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FINAL PROGRESS REPORT  
RESEARCH PROGRAM  
ON  
NOISE MITIGATION IN SONAR II

Contract NOnr - 142(00).  
Task Order NOnr - 142(02)

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CASE: 52-19-54

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October, 1962

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FINAL PROGRESS REPORT

RESEARCH PROGRAM  
ON  
NOISE MITIGATION IN SONAR II

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ABSTRACT

The design and performance of a comb filter used as an aid in aural detection of tones in noise are described. The comb filter has pass bands 31 cps apart, extending from 200 to 3500 cps. The width of the pass bands are adjustable down to 2 cps. The spectrum is aurally monitored for the presence of a tone while the signal is heterodyned with an oscillator whose frequency is varied linearly with time. The comb filter is synthesized using a delay medium with external electrical feedback. The delay is provided acoustically in air in a copper tube. Laboratory listening tests, of a 1-kc tone in white noise, show an improvement of 7.5 db in signal threshold when the comb filter is used, as compared with naked-ear detection. Spectrograms are presented (made from USNUSI recordings) showing the use of the comb filter in enhancing tones radiated by the USS TUSK and the USS HALFBEAK. Tests made at sea (Sub Dev Grp Two) aboard the USS HALFBEAK with the K-1 as a target are discussed.

An analysis of a system designed to detect the modulation of propeller cavitation noise is made. The method consists in square-law rectification followed by detection of a resulting tone at the modulation frequency. For a signal 6000 cps wide and 10 seconds long it is shown that a 5-db reduction in threshold over naked-ear detection may be attained. Additional improvement in threshold may be obtained by longer observation of the signal, but only 1.2 db is realized for every doubling of observation time.

A general search problem consisting of detection of a tone in noise in a given length of time is discussed. The frequency of the signal is assumed to be unknown, but between certain limits. The increase in signal threshold due to uncertainty of the signal frequency is evaluated for an ideal detection system.

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(The ideal detection system is taken as one consisting of a bank of contiguous band-pass filters of bandwidth equal to the reciprocal of the signal duration. The amplitude of the envelopes of the filter outputs are used as the basis of detection). It is found, for example, that the 50% threshold of detection for a system requiring thirty filters (to cover the range of possible signal frequencies) is 3-3/4 db higher than the threshold for the single-filter system which may be used when the signal frequency is known. Tests with aural detection are reported which show that the 50% threshold is 2-3/4 db higher when the signal frequency may be anywhere in the 200- to 11400-cps band than when the signal frequency is known to the observers in advance. Comparisons are drawn between aural thresholds and those attainable with the ideal system under the same conditions. A modification of the ideal system using fewer filters is found to be almost as good as the ideal system. A signal threshold 12 db lower than aural threshold may be realized with a modified ideal system in detection of a tone in a band of white noise 1.00 cps wide with five seconds of observation time.

Use of frequency multiplication followed by analysis with a single scanning filter as a substitute for the use of the great many filters required by the ideal system is discussed. A means of frequency multiplying using a recording medium, without changing the speed of the recording medium, is described, with application of this method to detection of tones in noise.

Oscillograms are presented showing detection of tones in noise with a frequency-scanning analyzer as a function of the signal-to-noise ratio. The parameters of scanning rate and filter bandwidth are discussed.

The dynamic range, and differential intensity sensitivity of a facsimile recorder are measured to determine its usefulness as a spectrum plotter.

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FINAL PROGRESS REPORT

RESEARCH PROGRAM  
ON  
NOISE MITIGATION IN SONAR II

PART I

SECTION A - INTRODUCTION

1. Background

1. This project is a continuation of the "Research Program on Noise Mitigation in Sonar", (Contract NOnr-142(00), Task Order NOnr-142(01), which was initiated in August 1950 and completed in November 1951 (Ref. 1). The objective of the original research program was to study means of lowering aural thresholds in passive listening systems. As a result of these studies a laboratory model of a comb filter was built which reduced the threshold of detection of tone signals in white noise by 6.8 db (Figure 1). A special volume expander was built for the enhancement of modulation on propeller cavitation. Experiments were made to investigate the use of frequency multiplication as a means of signal-to-noise improvement and to evaluate binaural listening as an aid in detection.

2. Purpose

2. The purpose of this project has been to continue work on the comb filter and build a model suitable for test at sea. The research phase has been continued, with a study of the detection of propeller cavitation (which has led to the development of a device for detection of modulation of propeller noise for the Bureau of Ships - Contract NObsr-52605), an evaluation of the effect on signal threshold of uncertainty in the signal frequency, and a determination of

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thresholds for certain ideal detection systems.

SECTION B - GENERAL FACTUAL DATA

1. Identification of Technicians

3. The names and experience of the technicians who have worked on this project are:

a. M. J. Di Toro

Division Head

Born: 1910

Education: EE, Polytechnic Institute of Brooklyn, 1931  
MEE, Polytechnic Institute of Brooklyn, 1933  
D. Eng., Polytechnic Institute of Brooklyn,  
1946

Licensed Professional Engineer, New York,  
New Jersey

Professional Experience:

1934-1941 Research Acoustical Engineer,  
Thomas A. Edison, Inc.

1941-1946 Sr. Electrical Engineer,  
Hazeltine Electronic Corp.

1946-1947 Assistant Director, Microwave Re-  
search Institute, Polytechnic In-  
stitute of Brooklyn

1947-June Division Head, Federal Tele-  
communication Laboratories.  
1952 Adjunct Professor of Electrical  
Engineering, Graduate School,  
Brooklyn Polytechnic Institute

b. W. Graham

Project Engineer

Born: 1924

Education: BS in ME, University of Wisconsin, 1944  
BS in EE, University of Wisconsin, 1947  
Graduate Work, Polytechnic Institute of  
Brooklyn 1947-present.

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Professional Experience:

1945 Instructor, radar, USNATTC,  
Corpus Christi, Texas  
1947 Instructor, mathematics, U. of  
Wisconsin  
1947-Date Project Engineer, Federal  
Telecommunication Laboratories

c. H. H. Anderson

Engineer  
Born: 1911  
Education: BS in EE Rutgers University, 1934  
MS in EE Columbia University, 1936

Professional Experience:

1934-1935 Gas Analyst, Standard Oil  
Co. of N. J.  
1936-1937 Mathematics Instructor, Board  
of Education, Elizabeth, N. J.  
1937-1938 Ass't. Instructor, Columbia  
University  
1939 Testing Electrical Devices,  
Underwriters Test Laboratory  
1940 Circuit Breaker Analysis, ITE  
1940-1941 Magnetic Mine Program Bureau  
of Ordnance, Navy Department  
1944-1945 Servo-Mechanism, Curtiss-Wright  
Corp.  
1945-Date Math. Analyses, Federal Tele-  
communication Laboratories

d. G. Greenfield

Junior Engineer  
Born: 1929  
Education: BEE, City College of New York, 1951

Professional Experience:

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1951-Date Jr. Engineer, Federal  
Telecommunication Laboratories

e. J. M. Pressler, Lieutenant, U.S.N.

Born: 1920

Education: BS, United States Naval Academy, 1944  
Special Weapons School, New London, Conn.,  
1947

BS in EE, U.S. Naval Post Graduate School,  
1951

Professional Experience:

1944-1949 Duty in Submarines

1946 Submarine qualification  
certificate

1952 Worked for three months at  
Federal Telecommunication Labora-  
tories in connection with duty  
at the U.S. Naval Post Graduate  
School

f. S. M. Schreiner

Sr. Engineer

Born: 1926

Education: BEE, Polytechnic Institute of Brooklyn, 1948  
Graduate Work - Polytechnic Institute of  
Brooklyn at present

Professional Experience:

1943-1944 Polytechnic Institute of Brooklyn,  
Laboratory Assistant

1946 USNATTC - Instructor in Radar

1947 Hazeltine Electronics - Technician

1948-Date Senior Engineer, Federal Tele-  
communication Laboratories

2. Time Statement

4. The hours spent by these technicians on this project were:

-6-

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M. J. Di Toro	285
W. Graham	1200
H. H. Anderson	460
G. Greenfield	1845
J. M. Pressler	576
S. M. Schreiner	405

3. Outside Activity and Contacts

5. On 24 October 1951, D. C. Lookingbill and R. G. Stephenson of the Sound Branch of NEL, and on 31 October 1951, M. E. Brady of the signal Enhancement Section of NEL, visited these Laboratories. On both of these occasions the work that had been done to date on this project was discussed.

6. On 29 and 30 October 1951, two members of this group attended the Fourth Navy Symposium on Underwater Acoustics at Willow Grove, Pennsylvania.

7. On 7 November. Lt. Comdr. E. D. Schrader and Mr. R. E. Ankers of BuAir and Mr. M. P. Duffy of Hazeltine Electronics visited these Laboratories. The significance to the Sonobuoy program of the work being done at Federal on signal-to-noise improvement and aural displays was discussed.

8. On 20 November 1951, Dr. E. V. Potter of BuShips visited these Laboratories and discussed the work being done on this project.

9. On 26 November 1951, W. Graham of these Laboratories attended the conference on Submarine Target Classification held by the Committee on Underseas Warfare of the National Research Council.

10. On 14 January 1952, Dr. M. J. Di Toro and W. Graham, of these Laboratories, and Lt. J. M. Pressler of the U. S. Naval Post Graduate School went to sea aboard the USS GROUPER from the New London Submarine Base to test

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the volume expander described in the Final Progress Report on the first phase of this contract - Task Order NOmr-142(01). The use of the volume expander was discussed with Capt. E. T. Hydeman and Comdr. J. T. Webster of Sub Dev Grp Two.

11. On 28-30 April 1952, the comb filter and the frequency multiplier developed on Contract NObsr-52605 were given preliminary sea tests aboard the USS HALFBEAK with the SSK-1 as a target.

12. M. J. Di Toro and W. Graham presented a paper, "Signal-to-Noise Improvement in Underwater Listening," at the Fifth Symposium on Underwater Acoustics which was held 5, 6 May 1952 at the Naval Research Laboratory. The paper will appear in the Journal of Underwater Acoustics.

13. W. Graham attended the Seventh Undersea Symposium held 15, 16, May 1952 at the Department of the Interior, Washington, D. C.

14. Further tests were made at USNUSL on 22, 23, May 1952, with tape recordings of USS HALFBEAK self-noise, and a range run of the USS TUSK. These experiments, and the sea tests are described below.

4. Acknowledgment

15. We wish to thank the personnel at various Navy activities for their co-operation in the course of this work; particularly, Dr. John Ide and Mr. W. F. Saars of USNUSL, Capt. E. T. Hydeman and Comdr. H. B. Sherry and Comdr. Mitchell of Sub Dev Grp Two, and Comdr. John Geyer of OMR.

SECTION C - DETAIL FACTUAL DATA

1. Detection of Tones in Noise, Using a Comb Filter

a. Design of Comb Filter

16. Fletcher (Ref. 2), who formulated the concept of "critical bandwidth," found that noise outside of the critical bandwidth was ineffective in masking a

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tone in the center of a "critical band", and that use of a filter narrower than the critical bandwidth lowered the threshold of such a tone. An improvement of about 2 db per octave of bandwidth reduction is realized.

17. The comb filter, as applied to aural detection of tones of uncertain frequency, is used to achieve the reduced thresholds possible with very narrow filters without the excessive increase in search time occasioned by the use of a single narrow-band-pass filter. The comb filter (see Figure 2) consists of a number of narrow-band-pass filters (teeth) which are spaced so that there is no appreciable masking of a signal in one pass band by noise in adjacent pass bands. The frequency range of the unit is 200-3500 cps. The time required to search a spectrum for a signal as compared to the time required for a single filter is reduced by the number of teeth in the comb filter.

18. A comb filter may be synthesized by the use of a delay line and external feedback (see Figure 3). Signal frequencies are fed back to the input of the delay line, but they are in phase with incoming signals only for those frequencies which are integral multiples of  $1/T$ , where  $T$  is the delay time of the line. Consequently, resonant peaks are established at those frequencies. The bandwidth of the teeth is determined by the net circuital gain ( $AK$ ) around the feedback loop.

19. The chief determinants in the design of the comb filter are the spacing between teeth, the bandwidth of the teeth, and the searching time.

20. The choice of intertooth spacing is a compromise between maximum signal-to-noise improvement and minimum scanning time, since placing the teeth closer together to reduce the scanning time eventually results in mutual masking. Where the critical bandwidth is 50 cps, tooth spacing of 25 cps is adequate since

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it places each tooth in the center of a 50-cps band. Where the critical bandwidth is wider, as above 600 cps, spacing as little as 25 cps would cause some masking; therefore, 31 cps spacing was used in the final model.

21. The bandwidth of the individual teeth should be made as narrow as possible to effect the maximum signal-to-noise improvement, subject also to limitations imposed by scanning time. Experience has shown that a minimum of three seconds should be allowed for detection in each part of the spectrum so that 18 seconds are required for scanning with tooth widths of 5 cps and inter-tooth spacing of 30 cps. In the comb filter built, the bandwidth has been made adjustable, down to about 2 cps over part of the spectrum. The bandwidth is varied by adjusting the amount of feedback "A" (see Figure 3). "A" may be increased to where the product AK is close to unity and the system goes into oscillation. The comb filter may be tuned manually for slower scanning rates. The relations among tooth width, peak-to-valley ratio, and loop gain AK are shown in Figure 4.

22. An acoustic delay line was used (Figure 5). A 37-1/2-foot length of 3/4-inch copper tubing was coiled into a cylinder 12 inches high and 12-1/2 inches in diameter. This pipe length at an ambient operating temperature of 20°C results in a delay of 32.2 milliseconds, giving a  $\frac{1}{0.032} = 31$ -cps spacing. The pipe was potted (Figure 6) to reduce acoustic radiation and pickup.

23. The drive unit is a University 4401-Tweeter, rated at six watts. The pickup unit is an Altec-Lansing 21b condenser microphone.

24. Because attenuation in the pipe varies as a function of frequency, (Ref. 3) the delay unit must be equalized. For a given bandwidth the amount of equalization (the difference in attenuation at the ends of the band) is less at

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higher frequencies than at lower frequencies. On the other hand the amount of attenuation in the pipe becomes prohibitive at very high frequencies. A compromise, using the band 4 to 8 kc, was made. The input signal (200-3500 cps) is heterodyned to this range by the modulator described in the next section. The frequency responses of the equalizer, and the unequalized and equalized delay line are shown in Figure 7.

25. The circuit of the delay line and feedback loop are shown in Figure 8. For the sake of gain stability, a voltage-regulated power supply and considerable negative feedback have been used in the loop determining the bandwidth of the teeth in the comb filter.

b. Spectrum-Translating Modulator

26. The function of the spectrum-translating modulator is to shift the input spectrum to the frequency range of the comb filter, and to scan the input spectrum back and forth before the teeth of the comb filter so that a tone in the input is certain to fall in one of the pass bands of the filter during part of the scanning cycle.

27. This is accomplished by modulating a variable frequency carrier with the input spectrum, selecting the upper side band, and demodulating with the same carrier after passage through the comb filter. The circuit of the comb filter and modulator is shown in block form in Figure 9 and its action in the frequency domain in Figure 10.

28. The modulation is accomplished by the use of double-balanced ring modulators, similar to those used in carrier-current communication systems (Ref. 4). When a carrier of frequency  $c$ , and a signal of frequency  $v$  are applied, the output contains components of the form  $c + v$ ,  $c - v$ ,  $c$ , and higher-order

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distortion products. The band-pass filter shown in Figure 11, is used to separate  $c + v$  from the other signals. The component  $c$  is small and is called the residual carrier leak. It is made small by proper proportioning of two resistances in the modulator to compensate for unbalance in the modulator elements. The distortion products are of the form  $c \pm 3v$ , and  $c \pm 5v$ , some of which fall in the selected band  $c + v$ . By a proper choice of operating frequencies  $c$ , the presence of these components can be minimized. This was one of the reasons for choosing 4 kc as a carrier frequency. (The reader is referred to IER Nos. 1 and 2, of this project for further details.)

29. The carrier for the modulators is supplied by an RC phase-shift oscillator, (Ref. 5) at a nominal frequency  $f_0$  of 4 kc. With the values chosen,  $f_0$  varies linearly with the change in a control resistance at the rate of 21 cps per kilohm, provided the total change is kept small. Actually, at this operating point, a range of several hundred cycles per second is possible without affecting the linearity appreciably. (See Figure 12.)

30. The method of varying the frequency makes use of a nonlinear element, made by General Electric, called Thyrite (Figure 13). This two-terminal device has a variational impedance which depends on the d-c control voltage impressed upon it. By a proper choice of operating points, a linear relation between control voltage and dynamic resistance is obtained. By making the control voltage of the form shown in Figure 12, a similar variation of oscillator frequency is obtained.

31. The control voltage is obtained by integrating the square-wave output of an astable multivibrator (period = 34 sec). The integrating network consists of a ten-megohm resistance and the input capacity of a 6AU6

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amplifier. The gain is about 24 and the capacity between plate and grid has been made one microfarad, resulting in an input capacity of 25 microfarads. The resulting 250-second time constant is roughly 14 times the half period of the square wave, resulting in good linearity.

32. Further details of the circuit of the oscillator and modulator can be found in Figure 14A. Most of the circuits are straightforward and need no explanation. However, a few words will be said about the variational impedance circuit. In order to induce a change of 3% cps, a change in resistance of 1620 ohms is required. The dynamic characteristic of an appropriate Thyrite sample was measured, and the operating point selected in accordance with the best possible linearity and greatest slope. The process of obtaining the proper driving voltage for the integrator and the proportioning of the cathode resistances were purely matters of experiment. The two-henry choke was added to prevent the 1800-ohm cathode resistor from affecting the total impedance of the variational system.

33. One more method of resistance variation was made available. Rather than use automatic scanning, the operator can switch to a manual control, the full range of which corresponds to the range of resistance variation of the Thyrite.

34. The circuit of the input stages of the unit is shown in Figure 14B. An inset on the drawing shows the components of the two filters and the modulator.

c. Mechanical Details of Comb Filter

35. The circuitry was mounted inside the cylinder formed by the delay line. The power supply was mounted on a circular plate at the bottom. The

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additional units were mounted on three chassis and placed vertically inside the pipe. Figure 15 shows a top view of the encased delay line and power supply. Brackets were installed along the wall of the pipe for the purpose of guiding the chassis into their connecting sockets. Figures 16 and 17 show additional views of the comb filter.

36. The plug-in units are shown more clearly in Figures 18, 19, 20, and 21. Separate terminal strips were used for all components.

d. Experimental Results

(1) Listening Tests - Tones in White Noise

37. The laboratory listening tests were run as follows. The observers heard twenty-five 10-second groups per test. Each group consisted of four seconds of noise alone, five seconds of noise with which a 1-kc tone had been mixed, and one second of silence. In the five-second interval, the tone was set at various levels of intensity. In seven out of the twenty-five cases no tone was sent, in order to check for commissive errors (a tone imagined where there was none).

38. The experimental arrangement for these tests is shown in block form in Figure 22.

39. The test situation differed from a practical search situation in the absence of the carrier-frequency variation. In order to insure that the tone frequency coincided with the center of one of the comb teeth, the noise was turned off between tests and the output of the filter peaked at the signal frequency by varying the carrier frequency.

40. Since the only significant noise energy is that contained in a critical band, a band-pass filter, centered at 1 kc, was used with a VTVM to

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metor the input to the comb filter. Periodic checks were made to insure that the rms noise level in that band and the tone level were set at an equal voltage with 45 db of tone attenuation as an arbitrary reference level.

41. The results of this test are shown in Figure 23. The dotted lines indicate one standard error at each signal level. At the 50 per cent RD level, an improvement of 7.5 db in threshold was noted. At 45 db ( $s/n = 1$ ), a 92 per cent RD was attained, for the naked ear.

42. However, it has been found experimentally that a 50 per cent RD results when the signal and noise energy are equal in a critical band. This discrepancy is explained by the fact that the critical band width at 1 kc is only 63 cps while the bandwidth of the measuring system was 86 cps.

(2) Tests With Recordings of Submarine Sounds

43. The comb filter was taken to USNUSL on the 22, 23 May 1952, where experiments were conducted with recordings made by the USNUSL Recording Section.

44. The results from a run of the USS TUSK taken aboard the USS FLYING FISH are shown in Figure 24. Actually the signals are detected aurally but for purposes of presentation spectrograms taken with analyzers of bandwidths comparable to that of the critical bandwidths of the ear serve to show the performance of the comb filter. For purposes of actual detection, the time required to take such a spectrogram with the equipment used would be prohibitive.

45. Spectrum A is taken directly from the recording with the analyzer bandwidth set at 20 cps. Since the bandwidth is less than the critical bandwidth of the ear any audible tones will show up in the spectrogram. The only peak of significant amplitude is found at 120 cps and this was identified as self-noise.

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In spectrum B the output of the comb filter is presented (with 20-cps analyzer bandwidth) with the comb filter tuned aurally to the 120-cps tone. It is seen that the tone is raised about 5 db more above the level of other components. In spectrum C the comb filter has been retuned aurally to bring out other tones, which, in this case are not audible without the comb filter. Two of these appear for the same setting of the tuning, at 490 and 800 cps. They are raised about 6 db above the general spectrum level.

46. Spectrograms taken from recordings of self-noise made aboard the USS HALFBEAK while hovering are shown in Figure 25. The spectrograms on the right, with a 50-cps analyzer bandwidth, are indicative of what may be heard. The spectrograms on the left, made with a 5-cps bandwidth, illustrate the improvement possible with narrow-band filtering. The upper spectrogram on the right shows possible tones at 340 and 370 cps. The spectrogram of the actual signal made with the 5-cps bandwidth shows tones at 120, 260, 370, and 520 cps. The two signals through the comb filter with the 5-cps analyzer show the individual teeth in the comb filter. In spectrum B the tones at 270 and 500 cps are apparent, and in spectrum C the tuning is such that the 370-cps tone is passed. In spectrum B with a 50-cps filter no tones are visible other than those detectable without the filter. In spectrum C, on the right, the 270-cps tone is raised above the noise by the comb filter.

(3) Test at Sea

47. During 28-30 April 1952, the comb filter was taken aboard the USS HALFBEAK with the K-1 acting as a target part of the time. With the K-1 snorkeling no tones were audible over the JT equipment since screw noises predominated. Therefore, the comb filter was not useful. When both boats were

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quiet, numerous tones were heard, using the comb filter, but the operators unable to separate self-noise from target sounds with certainty. The results were also influenced by the fact that the HALFBEAK operated below the thermocline for safety reasons. The conclusion drawn from these tests was that some type of recording device should be used to aid the operator in distinguishing his own boat noise from target sounds.

**2. Detection of Modulated Wide-Band Signals**

48. Work on the problem of detecting modulated cavitation noise during the first year of this contract centered around the use of a volume expansion technique. This process was based on the fact that the difference limen of the ear for noise intensity is about 0.4 db (Ref. 6) while the inherent fluctuations in the noise are only about 0.1 db for a 3000-cps band in an analogous electrical system having the same build-up time as the ear (Ref. 1, p. 19).

49. Therefore, it was felt that enough volume expansion would, by enhancing the depth of modulation, reduce the limiting factor in detection to the statistical nature of the noise rather than to the properties of the ear.

50. The volume expander built at these Laboratories used a square-law rectifier, followed by a postdetector filter straddling the expected band of modulation frequencies (1-15 cps). The output of the filter was used to vary the gain of the signal. Although facilitating turn count at high signal-to-noise ratios, no improvement was noted at signal threshold. Furthermore, in work done on this problem for BuShips (Contract NObor-52605,) no improvement was found over naked-ear threshold until the postdetector bandwidth was reduced from 15 cps to 1 cps. At 0.15 cps, an improvement of 6 db at 50 per cent RD was

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found (Ref. 7). This indicated that the output of the postdetector filter in the original volume expander contained too much noise to effectively vary the gain.

51. The success of the method, then, is limited by the possibility of deriving a signal at the modulation frequency. In fact, once having this signal, its use in the expander is only one of a number of ways of indicating the presence of a cavitating target.

52. The results quoted above led to the need for an analytical study of modulated cavitation noise to determine the extent to which rectification and narrow-band filtering could improve detection.

53. Briefly, the analysis consists in assuming Fourier series representations for both the target and background noises. Each carrier frequency of the target noise is assumed to be sinusoidally modulated to a depth of 100%. After square-law rectification all background carriers, target carriers, and target side bands combine to give sum and difference frequencies or direct-current and double-frequency terms. Of these the difference terms which fall around the modulation frequency are found both when the target is assumed present and when it is assumed absent.

54. When no target is present the signal output of the narrow-band-pass filter around the assumed modulation frequency has the properties of a white noise. When a target signal is present the filter output consists of a sine wave mixed in this noise. The statistical properties of the envelopes of these two signals are given by Rice and Bennett (Ref. 8), and from their curves the probability of detecting a target of any assumed strength relative to the background noise may be calculated. The method is shown in Appendix I.

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55. Some of the results of this analysis are shown in Figure 26. The probability of not detecting the presence of a target (one minus the Recognition Differential) is plotted versus peak signal-to-noise ratio. The parameters in these curves are as follows: the predetector bandwidth (the bandwidth of the target signal) is assumed to be 6,000 cps, the time of observation of the signal is assumed to be 10 seconds, the frequency of amplitude modulation of the target is assumed to be known ( $r = 0$ ) or is assumed to be known within 10 cps ( $r = 10$ ), the probability of judging a target to be present when there is none is equal to 0.05 (the probability of commissive error), and the postdetector bandwidth (the narrow-band-pass filter following square-law detection of the target and background) is assumed to be either 1/2 cps or 1/10 cps. An experimentally determined curve for aural detection is shown for comparison.

56. Comparing the curve for the ear with that for the 1/10-cps filter with the modulation frequency known (as it was in the aural test), it is found that the threshold for detection (50% point) with the rectification and filtering technique is 5 db lower than that for aural detection. It is seen that detection with a 1/2-cps filter is inferior to that with a 1/10-cps filter (the magnitude of this difference is discussed in Appendix I.)

57. When a 1/10-cps filter is used, the threshold of detection for the case in which the modulation frequency is uncertain (within 10 cps) is 2-1/2 db higher than for the case in which the modulation frequency is known. This factor is given further attention in a separate section in this report.

58. To summarize, it has been found that a 5-db improvement over aural threshold in the detection of modulated cavitation may be realized by the use of this detection method with postdetector filters of 1/10-cps bandwidth.

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Since screw and blade rates of cavitation may vary from about 1 cps to 15 cps, 140 fixed filters would be required to monitor the spectrum for the presence of a tone indicative of a target signal. Since this represents a prohibitive amount of equipment, frequency multiplication with aural detection was used (this equipment was built on Contract NObsr-52605) for analyzing purposes. This process consisted of recording the rectified signal on a slowly moving magnetic tape and picking up the signal with heads moving at high speed relative to the tape. The amount of multiplication used was 500 times, which translated the 1- to 15-cps band to 500 to 7500 cps. The average critical bandwidth of 50 cps is then equivalent to  $50/500 = 1/10$  cps.

3. Some Remarks on the General Search Problem

59. It is instructive to consider the general problem of detecting a tone in a white noise when the frequency of the tone is not known in advance. This is generally the case during search. The extent of uncertainty of signal frequency is always limited. In the case of underwater listening, the frequency range of interest is limited by attenuation in the water. In the case of detection of modulated cavitation the frequency range of the tones (which are generated by square law-rectification) is limited to usable shaft and blade rates.

60. Setting aside, for the moment, considerations of equipment complexity, the threshold levels for ideal detection systems may be computed and the performance of these systems may be compared with that of the ear. When the potential improvement is known, systems may be designed which compromise complexity with the degree to which the optimum is approached.

61. In order to do this, agreement must be reached as to what constitutes an optimum system. Van Vleck and Middleton derived the so called "matched

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filter" (Ref. 9) (using peak output S/N as a criterion) which is a filter having an amplitude response proportional to the amplitude of the spectrum of the signal, and a phase response the negative of that of the signal spectrum. For purposes of this analysis an approximately Gaussian-shaped filter centered at the frequency of the signal with a bandwidth of about the reciprocal of the duration of the signal is used. The assumption is made here that the optimum thing to do when the frequency is not known is to use a bank of such filters to cover the frequency range of interest.\*

62. The basis of detection is taken as the amplitude of the envelope of the outputs of the filters. Then the work of Rice and Bennett, (Ref. 8) Figures 39 and 40, may be applied directly to find the probability of detecting a signal. A given probability of commissive error (judging a target present when there is none) is assumed, and this determines a bias level which the signal must exceed in order to be detected. Sample calculations are given in Appendix I.

63. The results presented in Figure 27 indicate the increase in signal level which is required for the same probability of detection when the signal frequency is not known as compared with when the signal frequency is known. The detection system consists of a number,  $N$ , of contiguous band-pass filters of bandwidth  $1/T$ , where  $T$  is the signal duration. When  $N = 1$ , the signal frequency is known, and the signal-to-noise ratio is +4 db at 50% probability of detection. If  $N = 100$ , however, the signal-to-noise ratio must be +8-1/2 db for the same probability of detection. In less general terms this means that

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\*The effective noise bandwidth of a Gaussian filter is very nearly equal to its 3-db bandwidth. Adjacent filters are assumed to cross over at the 3-db points.

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a pulse of duration 0.2 seconds could be detected at +4 db in a filter of 5-cps bandwidth if its frequency were known; but if it were only known that the frequency were within some 500-cps band, then the signal would have to be +8-1/2 db above the noise in a 5-cps band to be detected (with 50% probability).

64. It is interesting to compare these results with the results of tests with aural detection of tone signals for the cases in which the listeners knew the signal frequency in advance, and those in which the listeners were uncertain of the signal frequency (within 1200 cps) (Figure 28). The manner in which these tests were conducted is described in Appendix II. At 50% probability of detection there is a difference of 2-3/4 db between the means for the two conditions. If the ear is likened (as it often is) to a bank of contiguous filters of widths equal to the critical bandwidth then there are approximately 30 such critical bandwidths in the 1200-cps band investigated (200-1400 cps). Referring to Figure 27, it is seen that an ideal system, in which only one observation of the filter outputs is made, requires 3-3/4 db greater signal for thirty filters than for one. There is approximate numerical agreement with the experimental test results. The theoretical and experimental thresholds may be expected to differ because the ear is analogous to, but not identical with, a bank of filters. Furthermore, in the aural tests the observation time was five seconds, which allows many observations, and confirmations, of the presence of a signal.

65. In Figure 29A curves are presented which permit a direct comparison between the performance of the ideal system discussed above, and the measured performance of the ear in detection of tones in noise. Curves for a

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modified form of the ideal system are also given. The modified system consists of a bank of filters each of which is wider than the optimum bandwidth ( $1/T$ ). Since these filters are wider than the optimum, they build up faster. Therefore, more observations of their outputs may be made in a given time and the signal is judged present or absent on the basis of whether a majority of the outputs of each filter exceed the bias. For the sake of generality the computed curves have been presented in terms of a number of parameters.

These are:

$T_0$  = the time of signal observation for the reference system,  
seconds

$c_0$  = the bandwidth of one of the ideal filters, ( $1/T_0$ ),  
for the reference system, cps

$T$  = the time of signal observation

$c$  = bandwidth of filters, cps

$r$  = frequency uncertainty of signal, cps

$H$  = number of observations in time  $T$   
( $H = 1$  for ideal system)

66. A summary of the results presented in Figure 29A is given in the table below for the special case in which the frequency of the tone is known only to fall in a 1200-cps band. The difference between the 50% detection threshold for each system and that of aural detection is noted.

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TABLE I

Comparison of Aural Detection of Tones with Detection  
Using a Bank of Contiguous Band-Pass Filters  
(tone in a noise 1200 cps wide)

Curve in Figure 29A	Time of Observation (sec)	Number of Filters	BW of Filters (cps)	No. of Obs. of Filter Outputs	Improvement Over Aural Detection (db)
G	5	Err			0
A	0.2	240	5	1	2
B	0.6	240	5	3	4-1/2
C	1.0	240	5	5	6-1/2
D	0.6	720	1-2/3	1	6-1/4
E	1.0	1200	1	1	8-1/4
F	5	1200	1	5	12

67. Consider the reference system of curve A in Figure 29A. This curve applies to any system consisting of 240 contiguous filters where the bandwidth of the individual filters equals  $1/T$ . For example,  $T$  may be one second,  $c = 1$  cps, and  $r = 240$ ; or, as in Table I,  $T = 0.2$  seconds,  $c = 5$  cps, and  $r = 1200$  cps. The 50% threshold is 15 db below the level of the noise in  $r$  cps. (Referred to the noise in one of the filters the 50% threshold is  $-15 + 10 \log 240 = +8.8$  db. This is the same curve plotted in Figure 17 for  $N = 240$  where the reference bandwidth is taken as the noise in one of the filters.)

68. Looking at curve G, the ear (with  $r = 1200$  cps but  $T = 5$  secs, taken from Figure 26) is found to be about 2 db worse than system A. The systems of curves B and C require no more apparatus than that of curve A,

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but are drawn for three and five observations of the same filter outputs ( $T = 3T_0 = 0.6$  sec and  $5T_0 = 1.0$  sec). These systems are  $4\frac{1}{2}$  and  $6\frac{1}{2}$  db better than the ear, respectively, and take considerably less time to scan a particular signal (bearing).

69. Curves D and E are drawn for ideal systems, which for the case of  $r = 1200$  cps, gives  $T = 0.6$  and 1.0 seconds, and  $c = 1\frac{2}{3}$  cps and 1 cps respectively. These systems are about  $6\frac{1}{4}$  db and  $8\frac{1}{4}$  db better than the ear, respectively. The final curve, F, is for a modified ideal system using five times as many filters as the system of curve A, and five observations of the outputs of these filters. This makes  $T = 25T_0$ , which for the example being discussed is 5 seconds - the same as in the aural experiment (curve G). This system is 12 db better than the ear.

70. It is interesting to compare the ideal systems (curves D and E) with the modified ideal systems (curves B and C). Curve D (720 filters) is just  $1\frac{3}{4}$  db better than curve B, which represents three observations on 240 filters. Similarly curve E (1200 filters) is only  $1\frac{3}{4}$  db better than curve C for five observations of 240 filters.

71. The improvement in threshold which may be realized by taking a majority report of a number of observations of the filter outputs is shown in Figure 29B, for systems of one and 240 filters. It is seen that it is relatively more advantageous to make more observations when the number of filters required is greater.

72. The question arises as to whether it is desirable to filter at all, since the majority report of many observations gives substantial improvements and would require less equipment. In Figure 30 the ideal system is compared with

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a system using only one filter of bandwidth equal to the frequency uncertainty of the signal,  $r$ . In the upper half of the figure, the probability of detecting a signal is plotted against the signal-to-noise ratio in the band,  $r$ , with  $Z$ , the product of the  $r$  and the time of observation  $T$  as the parameter. Either  $r$  or  $T$ , may be considered fixed. For example, if  $r = 100$  cps,  $T$  takes on the values 1, 2, 3, 5, 10, and 20 seconds corresponding to the parameters  $Z$  of one hundred times those values. A consistent improvement of about 2.7 db is noted for every doubling of the time of observation, and the number of filters. If  $T$  is considered fixed, say at one second,  $r$  takes on 100, 200, etc. For  $r = 100$ , 50% threshold obtains at -11.5 db compared to the noise in 100 cps. With the noise in one of the 100 filters as reference (each of bandwidth  $1/T = 1$  cps) the signal-to-noise ratio is  $-11.5 + 20 = +8.5$  db. (This corresponds to the point on the curve  $N = 100$  in Figure 27.)

73. The lower curve shows what is theoretically possible with a single filter using majority report. (The method of computing these results, and the assumptions made, are given in Appendix III.) The parameter  $N = rT$ , represents the number of observations of the output envelope of the filter which may be made in time  $T$ . Thresholds for  $N = Z$  should be compared. For example, let  $r = 100$  and  $T = 1$ . The ideal system has a threshold of -11.5 db. The single-filter system has a threshold of about -6.5 db, or about 5 db worse. For  $r = 100$  and  $T = 10$  seconds,  $N = Z = 1000$ , the difference between the two systems is 5 db. For  $N = Z = 1$  the systems are the same. It is seen that as the time of observation is increased the ideal system becomes progressively better than the single-filter system by roughly 5 db for every ten times increase in time.

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74. If the signal is not a tone, but an amplitude-modulated noise from which a tone is derived by square-law rectification, the rate of threshold improvement with increase in observation time is not as great as when the signal is a tone to start with. This may be viewed as a loss due to intermodulation products between signal and noise in the square-law rectification. The thresholds of detection of 100% sinusoidally amplitude-modulated white noise mixed in a white-noise background are shown in Figure 31 as a function of the time of observation. The bandwidth of the signal and background are assumed to be 6000 cps, and the ideal system described above is used to detect the tone at the modulation frequency. The uncertainty of the modulation frequency is taken as 10 cps which is approximately the uncertainty to be expected with propeller cavitation. It can be seen from the figure that doubling the time of observation (and the number of filters) results in about 1.3-db threshold improvement whereas 2.7 db was realized when the signal was a tone to start with.

75. In this section the impracticability of using large numbers of filters has been overlooked while the potential improvements in threshold have been investigated. In the following section a practical method for signal analysis is presented, which achieves the frequency resolution desired, in the shortest possible time, with a reasonable amount of equipment.

4. Spectrum Analysis by Frequency Multiplication Followed by Filtering

a. Theory

76. As discussed in the section on signal analysis, the problem of finding a tone of an uncertain frequency in a noise may be viewed as one of obtaining plots of the power spectra of incoming signals. When such a spectrum

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exhibits a peak, a tone may be said to be present with an arbitrarily small probability of being wrong (the latter depends on how large a peak is required to be judged a tone).

77. When the frequency uncertainty of the tone is large compared with the resolution required, conventional systems of spectral analysis become uneconomical (banks of contiguous filters with scanning of their outputs), or too time consuming (a single filter used to monitor the entire spectrum by heterodyning). For example, consider the analysis of a band of noise 1200 cps wide with a desired resolution of 5 cps. Two hundred and forty fixed filters, with commutating means for their outputs, would be required if contiguous filters are used, with a time for analysis of the order of 0.2 secs (the build-up time of one of the filters). If a single 5-cps filter were used analysis would take 48 secs, allowing 0.2 secs for the filter to build up in each 5-cps band.

78. However, there is another way of analyzing the spectrum with a single filter without increasing the time for analysis over that required for a contiguous-filter analyzer. The method consists of frequency multiplying the signal prior to analysis, followed by scanning with a single filter by the heterodyning technique. An analyzer of this type has been described by Mathes, Norwine, and Davis in an article, "Cathode Ray Sound Spectrograph" (Ref. 10). In this system the signal was first recorded on a disc (2 seconds); the disc was then speeded up 200 times (7 seconds); the signal was then analyzed (11 seconds, but 1/2 second would suffice for detection of a tone); and then the disc was slowed down (16 seconds). Since it takes much longer than the duration of the signal to analyze the signal with this system it would not be very useful for signal detection purposes (nor was it designed for those purposes). A modification of this system, however, will permit continuous

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signal analysis. This is accomplished by multiplying, without changing the speed of the recording disc, in a manner described below. In this system one signal sample is analyzed while the next sample is recorded for subsequent analysis.

79. Before considering the details of the proposed system it is desirable to find the minimum factor of multiplication which will enable the analysis to keep up with the flow of information. The symbols used in this discussion are as follows:

$W$  = bandwidth of the signal to be analyzed, cps

$T$  = duration of the signal, seconds

$b$  = maximum frequency resolution possible, cps ( $b = \frac{1}{T}$ )

$t$  = time required to analyze the spectrum with a single filter  $b$  cps wide, seconds

$N$  = factor of frequency multiplication

$k$  = a factor equal to the ratio of the actual time spent in any bandwidth  $b$ , to the nominal build-up time  $1/b$ .

80. In each bandwidth  $b$  cps,  $k/b$  seconds are spent. There are  $W/b$  bands  $b$  cps wide in  $W$ . The time  $t$  required to scan  $W$  with a filter  $b$  cps wide is then:

$$t = (W/b) \times (k/b) = kW/b^2. \quad (1)$$

After frequency multiplication of  $N$  times, the bandwidth of the signal becomes  $W' = NW$ , the duration becomes  $T/N$ , and the maximum resolution (the smallest analyzing bandwidth) becomes  $b' = Nb$ . The new scanning time  $t'$ , assuming that the signal is repeated until the analysis is completed, is:

$$t' = (W'/b') \times (k/b') = kW'/b'^2 = kW/(Nb)^2 = kW/Nb^2. \quad (2)$$

Therefore,  $t'$  is equal to  $t/N$ . If  $t'$  equals  $T$ , the analysis can keep up (with

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a delay of  $T$  seconds) with the flow of signal information. Setting  $t' = T$ , which makes  $N$  a minimum:

$$t' = kW/Nb^2 = T. \quad (3)$$

Eliminating  $b$  by substituting  $T = 1/b$ ,

$$N = kTW. \quad (4)$$

81. Applying this to the example considered above, in which a 1200-cps band was to be analyzed with 5-cps resolution:

$$T = 1/b = 1/5 = 0.2 \text{ seconds.}$$

Let the parameter  $k = 1$  (the choice of  $k$  will be discussed below).

$$N = kTW = 0.2 \times 1200 = 240$$

$$W' = NW = 240 \times 1200 = 288 \text{ kc}$$

$$b' = Nb = 240 \times 5 = 1200 \text{ cps}$$

$$t' = kW/Nb^2 = 1200/240 \times 25 = 0.2 \text{ seconds} = T$$

82. Take as another example the detection of modulated cavitation signals by square-law rectification and search for a tone at the modulation frequency. The uncertainty of the signal frequency  $W$ , is about 15 cps. Require a resolution of 1/20 cps, so that  $T = 20$  seconds. Then:

$$N = kTW = 20 \times 15 = 300$$

$$W' = NW = 300 \times 15 = 4,500 \text{ cps}$$

$$b' = Nb = 300 \times 1/20 = 15 \text{ cps}$$

$$t' = kW/Nb^2 = 15 \times 400/300 = 20 \text{ seconds} = T$$

b. Utilization of This Method

83. As pointed out above, a practical detection device must analyze the signal at the rate it is received (with a delay), so that time may not be lost in accelerating and decelerating a recording medium. This may be accomplished

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by sampling the signal with very narrow pulses (pulse amplitude modulation) which are recorded on a drum so that they appear on playback to be closer together than the actual elapsed time between pulses. For example, consider that the sampling pulses are synchronous with the rotational speed of the drum, and that one pulse is recorded per revolution. Then subsequent pulses would be recorded on top of one another and on playback a single pulse would be read off. If the pulse rate is slightly different than the rotational speed of the drum, the pulses will be recorded close together and in sequence. In this way many revolutions of the drum are required to record the signal but the entire signal will be played back in a single revolution. The amount of frequency multiplication  $N$  is equal to the number of revolutions during recording in this case.

84. The design of a system based on this method will now be discussed, for the analysis of a signal of duration  $T = 20$  seconds, and bandwidth  $W = 15$  cps. The minimum multiplication required for ( $k = 1$ ) is 300 times, as shown above. If this signal is to be represented by pulse samples these samples must be taken at a rate of at least twice the highest frequency (Ref. 11), or 30 cps in this case. Since 30 cps is a convenient synchronous-motor speed, assume the recording disc is driven at this speed and that one recording head is used. In 20 seconds,  $30 \times 20 = 600$  pulse samples of the signal must be taken, and these are to be recorded around the circumference of the disc. Experience in the manufacture of magnetic pulse-storage systems for computers has shown that 100 pulses may be recorded per inch. (The mechanical tolerances required to achieve this are discussed by Wallace, Ref. 12.) Therefore, the circumference of the disc must be six inches to accomodate 600 pulses. The

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peripheral velocity of the disc will equal  $6 \times 30 = 180$  inches per second. The time between pulses on playback (which are 0.01 inches apart) will be  $0.01/180 = 0.0000555$  seconds. The pulse samples when recording must be taken slightly faster than 30 cps (every 0.0333333 seconds) so that they are spaced by 0.01 inches. The time between pulses samples is then  $0.03333 - 0.0000555 = 0.032778$  sec, giving a pulse-sample rate of 30.050 cps.

85. Since the entire signal will be played back in one revolution (1/30 sec) and the actual duration of the signal was 20 seconds, the resulting frequency multiplication is 600 times - or twice the minimum required.

86. The design of a system to handle a signal of 1200-cps bandwidth and duration 0.2 seconds is fundamentally the same as the previous example. However, it is not feasible to rotate the recording disc at twice the highest frequency of the signal. It is necessary to use a number of equally spaced recording heads around the disc so that a point on the circumference moves only the distance between two adjacent heads between signal pulse samples, rather than an entire revolution. In this case six recording heads would be used, and the drum would be rotated at 400 rps.

c. Choice of the Build-Up Parameter "k"

87. In a system which analyzes a spectrum with a single filter by varying the tuning of the filter, or by sweeping the signal frequency through the range of a fixed filter, the question arises as to how fast an analysis may be accomplished. This problem has been treated in the literature. Barber (Ref. 13), for example, analyzed the case of a simple RLC type of filter with a linear rate of frequency scan. It is found that as the rate of scan is increased the frequency at which the output of the filter is maximum becomes

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displaced from the actual signal frequency in the direction of the scan. Furthermore, the filter output builds up less and less, the response becomes broader (in per cent of the filter bandwidth), and the maximum response is followed by increasing oscillations as the scanning rate is raised. The magnitude of these effects is such that twenty times the build-up time of the filter ( $k = 20$ ) is allowed in scanning between the cutoff frequencies of the filter in spectrum-analyzing systems requiring high resolution in frequency, and wide dynamic range. However, for detection of a tone in a white noise the only important requirement seems to be that the output of the filter builds up close to the maximum value (steady-state response). This fact is borne out by the experimental results described below.

88. A Hewlett Packard Model 3COA Wave Analyzer was driven through a gear train, sprockets, and chain by a motor in a Sound Apparatus Company Pen Recorder. Measurement of the change in tuning with time of this combination showed that a fairly constant rate of 154 cps/sec obtained between 7400 and 9400 cps. In the first experiment an input tone at 8500 cps was used. The analyzer was run from 7800 to 9200 cps. Oscillograms of the responses were taken for different filter bandwidths (the bandwidth of this analyzer is adjustable). The results are shown in Figure 32, where the build-up parameter  $k$ , the ratio of the actual time spent in scanning between the cutoff frequencies (3-db points) of the filter to the reciprocal of the filter bandwidth, is given for each case. The percentage of steady-state response is also shown. These measurements are in close agreement with the results of Barber (Ref. 13). It is seen that with  $k = 1.46$  the output builds up to within 0.5 db of the steady-state response.

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89. The next series of experiments permit evaluation of the scanning analyzer as a tone detection system. The object of these measurements was to compare the thresholds obtained with a scanning analyzer with those calculated for the ideal detection system and to find the minimum usable  $k$ .

90. In the experiment the scanning filter took much longer to complete the analysis than would be required by the ideal system with many contiguous filters, but it is assumed that the analysis time can be reduced to the actual signal duration by frequency multiplication as described above.

91. Detection with the analyzer bandwidth set at 15 cps, giving  $k = 1.46$ , is illustrated in Figure 33. Four signal-to-noise ratios are shown, all measured with respect to the noise in 15 cps. The oscillogram covers a range of 1400 cps, or 93 times the analyzer bandwidth. The results may be compared with an ideal system with  $r = 1400$  cps,  $T = 1/14$  sec,  $c = 14$  cps, and  $N$  (the number of filters) = 100 (see Figure 27). The 50% threshold for the ideal system is +8-1/2 db.

92. In Figure 33 the bias level has been set so that it is not exceeded by the noise in any of the oscillograms since there are not enough measurements to find the proper bias to give excesses in 5% of the oscillograms. It is seen that at +15-db signal-to-noise the bias is exceeded by the signal in every case, at 10 db in half the cases, at 5 db in one quarter of the cases, and at 0 db not at all. (The frequency at which the signal appears varies because adequate care was not taken in synchronizing the oscilloscope with the wave analyzer.) The measured thresholds are within a few db of the ideal system.

93. The results of the same experiment repeated with the analyzer

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bandwidth set at 36 cps are shown in Figure 34. The signal-to-noise ratio is measured with respect to noise in the 36-cps band in these measurements. Therefore, 0 db in this figure would be equivalent to +3.8 db in Figure 33. There is an obvious difference in the appearance of oscillograms of Figures 33 and 34. Those taken with a 36-cps bandwidth have broad maxima at the signal frequency as compared with the maxima for a bandwidth of 15 cps which show narrow peaks. The reason for this difference is that the analyzer remains tuned for a relatively long time around the signal frequency with the 36-cps bandwidth (in fact for 8.45 times the buildup time of the filter). As a result, a number of observations may be made at the signal frequency in a single sweep. The bias, as drawn in Figure 34, is set so that no noise peak exceeds it. Fifty per cent threshold is between 5 db and 10 db. If, however, the bias is reduced so that single noise peaks exceed it, but four or more noise peaks close together do not exceed it (in other words, a majority report is taken), 50% threshold is not more than 5 db. The system of Figure 34 may be said to be equivalent to an ideal system with  $1400/36 = 39$  filters, with about eight observations of the filter outputs. Fifty per cent threshold for an ideal system with single observation and 39 filters is about +8 db (Figure 27). Majority report of eight observations of the ideal system would improve the threshold by about 4 db (see Figure 39B), making the 50% threshold about +4 db - which is about the performance of the system of Figure 34 taking majority report.

94. The results shown in Figure 35, with the analyzer bandwidth set at 21 cps, are intermediate between the two just discussed. Since  $k = 2.86$ , two or three observations may be made at the signal frequency in a single sweep. If the bias is reduced below that shown, and the criterion for signal

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present is changed from a single excess to two adjacent excesses, the 50% threshold becomes less than 10 db.

95. These experiments demonstrate the fact that for purposes of tone detection in white noise a scanning rate which allows slightly more than the nominal build-up time of the analyzing filter is adequate. They also indicate that the narrowest possible filter is the best to use, but a broader filter is almost as good (for the same over-all scanning time) if proper use is made of its output (majority report).

##### 5. Facsimile Recorder for Frequency-Intensity-Time Display

96. In previous discussions of listening tests, a need was expressed for a visual presentation which records the spectra of received signals. A three-dimensional portrayal may be used to meet this requirement which represents frequency and time on two mutual perpendicular axes, and amplitude in terms of line density. (Work on three-dimensional portrayals of speech sounds has been done by Bell Laboratories and is described in Ref. 14.)

97. Figures 36A and 36B show the details of a Western Union Facsimile Recorder which has been considered for this display. The recording medium is electrosensitive "Eleedeltoe" paper, which is marked in response to the current passed through it. The blackness of the mark is proportional to the current through the paper. The stylus shown in Figure 36A travel across the paper on an endless metal belt. At the same time, paper is moved past the belt by a friction drive. In Figure 37A are shown the results of impressing a 1-kc sine wave on the recorder. The amplitude of the sine wave has been changed in 2-db steps from "just visible" to maximum blackness. The upper limit of intensity is limited by two factors. First, at high current levels a severe arc is

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formed, charring the paper. Secondly, the results show that while a change of 2 db at low intensity levels can easily be detected, a similar change at high intensities is not nearly as evident. This indicates a nonlinearity in the response, limiting the dynamic range of the medium. The estimated usable range of the paper is 10 db.

98. In the lower part of Figure 37A is shown a recording of band limited noise (40 cps at 1 kc). The noise has a granular appearance.

99. If a sine wave is present in the noise, the result, as the analyzer sweeps through the frequency of the sine-wave, will be an increase in energy in the output of the filter. This results in an increase in line density on the recording paper. To simulate this effect, a pulsed sine wave was added to the noise. The repetition rate of the pulse was five seconds, corresponding to the sweep time of one stylus across the paper. The duration of the pulse was set at 25 milliseconds, corresponding to the build-up time of the 40-cps filter. The duration of the pulse is 1/200 of the sweep time so that the presentation is equivalent to a scan of the outputs of 200 filters. The results, in Figure 37B, show that the 50% threshold signal-to-noise ratio is about 7 or 8 db for four adjacent sweeps. There is loss of about 1 db in using this type of presentation as compared with that of Figures 33-35, because of the 2-db differential sensitivity of the paper. The optimum recording level is that which makes noise alone barely mark the paper.

SECTION D - CONCLUSIONS

100. The ccm filter effects an improvement in tone signal thresholds of about 7 db in both laboratory measurements, in which the tones are immersed

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in white noise, and in tests with recordings of actual submarine sounds. Seventeen seconds are required to scan the signal on any bearing for the presence of tones. Because of confusion due to the presence of tones in the self-noise of submarines a spectrum recording device would be a useful aid to the operator. The comb filter, as synthesized using acoustic delay, occupies a cylindrical volume 12 inches in diameter and 18 inches high. Seventeen vacuum tubes are used.

101. An analysis of detection of modulation of white noise indicates that a threshold 5 db lower than aural threshold may be realized by square-law rectification of the signal followed by filtering of the resulting tone at the modulation frequency. (A frequency multiplier has been built for the Bureau of Ships - Contract N0bsr-52605, which permits aural detection of the tone due to propeller modulation.)

102. The general analysis of the problem of detection of a tone in noise shows that thresholds as much as 12 db lower than aural thresholds (for the same time for detection) may be realized by an improved detection system. This system consists of frequency multiplication of the signal followed by analysis with a single scanning filter. The design of such a system is within the scope of present practice. Detection by this system may be made less dependent upon the operator's attention than aural detection. Experiments indicate that a time equal to one and one-half times the nominal build-up time of the scanning filter in such an analyzer is adequate for this application.

PART II - FUTURE OF THIS PROJECT

103. The initial goal of this project was to build a device to lower the threshold of detection of tone signals. The object of this work was to

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extend the range of detection of passive listening equipment. The comb filter, which lowers tone signal thresholds by about 7 db, is a realization of this objective. However, because of the great advances which have been made in this field in the last two years, improvements of this magnitude are not important to a quiet submarine listening to a snorkeling, or surfaced, submarine. The opinion of personnel at Sub Dev Grp Two is that the comb filter may find application in quiet vs. quiet submarine or snorkel vs. snorkel submarine detection. It should be evaluated for those purposes.

104. Work on the detection of cavitation noise is continuing on Bureau Ships Contract N0bsr-52605. Design of a spectrum analyzer (for the detection of cavitation) using frequency multiplication and a scanning filter will be done on this project.

PART III - SUPPLEMENTARY DATA

SECTION A - APPENDIXES

1. Detection of an Amplitude-Modulated White Noise (Propeller Cavitation)  
Immersed in a White-Noise Background

105. The signal due to a cavitating submarine consists of an amplitude-modulated noise which is immersed in background noise (a combination of ambient and self-noises (see Figure 38). The analysis rests on the following assumptions.

a. The spectra of the target sound and background are both flat with frequency, extending from some low frequency cutoff, say 500 cps, to  $500 + b$  cps.

b. The long-time averages of target and background intensities do not change, (i.e., are constant average-rms noises), for periods of 10 seconds.

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c. The target sound is modulated sinusoidally to a depth of 100 per cent (it is easy to make corrections for smaller depths of modulation).

d. The frequency of amplitude modulation (the blade rate) does not change by more than one per cent in 10 seconds.

106. The noises, due to both target and interference, are each represented in a Fourier series consisting of equal-amplitude, randomly phased components (carriers). Each target-noise carrier is modulated sinusoidally at the blade rate. The signal, consisting of target and interference noise, is then square-law detected, producing voltages due to difference frequencies (as well as other components which are eliminated by filters).

107. The low-frequency (difference frequency) voltage is filtered with a band-pass filter, the postdetector filter, of bandwidth  $c$  cps, at the modulation frequency. The envelope of the output of the filter is detected by a linear detector. The probability density of this detected envelope is found for the two cases in which the target signal is assumed absent or assumed present. A bias voltage is then selected so that when the target is not present this bias is exceeded with only a small probability (perhaps 0.05 - this is called the probability of commissive error). The probability that this bias is not exceeded when the signal is present (the probability of omissive error) is then found with the same bias. Systems having lower probability of omissive error for a given probability of commissive error and a given time of observation are then superior to systems having greater probability of omissive error.

108. The symbols used in the analysis are as follows.

$w_n$  = average noise background power, watts per cps

$w_s$  = average target power, watts per cps

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b = bandwidth of target and background noises, cps  
c = postdetector bandwidth, cps  
 $e_n$  = rms voltage of each carrier in the Fourier series representation of the background noise  
 $e_s$  = rms voltage of each carrier in the Fourier series representation of the target noise  
 $\delta$  - spacing between adjacent carriers, cps  
 $f_m$  = modulation rate, cps

109. The rms voltage of the background noise carriers,  $e_n$ , is related to  $w_n$  by:

$$\frac{e_n^2}{\delta} = w_n, \text{ so that } e_n = (w_n \delta)^{1/2}. \quad (1)$$

110. Each of the target carriers of rms  $e_s$ , if modulated 100 per cent, has two side bands of half its own amplitude. In a one-cycle-per-second band the target power,  $w_s$ , is then  $\frac{e_s^2}{\delta} + \frac{2e_s^2}{4\delta}$ , where the first term is due to carriers and the second term is due to the side bands. Therefore,  $\frac{e_s^2}{\delta} = 2w_s/3$ , and  $e_s = (2w_s \delta/3)^{1/2}$ , and each side band has a rms amplitude  $e_s/2 = (w_s \delta/6)^{1/2}$ .

111. Consider, now, square-law detection of the background noise alone. Take any two noise carriers which are  $f_m$  cps apart, (e.g.,  $\sqrt{2w_n \delta} \cos(w_n t + \phi_n)$  and  $\sqrt{2w_n \delta} \cos(w_n + m t + \phi_n + m)$ ). On squaring, aside from the square of each term which is at twice frequency, a cross-product  $2(2w_n \delta) \cos(w_n t + \phi_n) \cos(w_n + m t + \phi_n + m)$  results. When this is separated into sum and difference components, the latter, which is at  $f_m$  cps, is found to be  $2w_n \delta \cos(w_m t + \phi_n - \phi_{n+m})$ .

112. In a bandwidth b cps, there are  $b/\delta$  noise carriers, and  $f_m/\delta$  less than  $b/\delta$  cross-products between carriers separated by  $f_m$ . For b large

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compared to  $f_m$ , as in the practical case, there are nearly  $b/\delta$  cross-products. Each cross-product results in a voltage at  $f_m$  of amplitude  $2w_n\delta$ , and of random phase. These voltages therefore add as powers, so that the average power at  $f_m$  is given by  $(b/\delta)(2w_n^2\delta^2)$ . At frequencies within a few cps of  $f_m$  the average output is substantially the same because the number of cross-products is very nearly the same. If the band-pass filter, following the square-law detector, has an effective width of  $c$  cps, then the average power output of this filter with background noise input will be:

$$(c/\delta)(b/\delta)(2w_n^2\delta^2) = 2bcw_n^2, \quad (2)$$

corresponding to an rms voltage  $(2bc)^{1/2} w_n$ . The probability density function of the envelope of this voltage (found by a linear detector) is given in Figure 39 with  $a = 0$ . (The ratio  $a$  will be defined below.) The integrals of the probability density functions of Figure 39 are plotted in Figure 40 (Ref. 8).

113. When a target is present the average output power increases due to six additional sources of power at frequencies around  $f_m$  cps in the band-width  $c$  cps. These are:

(1) Power due to cross-products (beats) between carriers of the target noise. The average value of this power, found in the same way as that due to background noise, is  $\frac{b}{\delta} \times 4 \times \frac{4w_s^2}{9 \times 2} \delta^2 \times \frac{c}{\delta} = \frac{8}{9} bcw_s^2$ .

(2) Power due to beats between target carriers and background carriers. This power is  $\frac{4}{3} bcw_n w_s$ .

(3) Power due to beats between target side bands and background carriers. This power is one half of (2) above, or  $\frac{2}{3} bcw_n w_s$ .

(4) Power due to beats between target carriers and target side bands which are not side bands of these same carriers (but are side bands of

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carriers  $2f_m$  cps away). Each carrier has two such nonassociated side bands. The power due to this source is one-half of (1) above giving  $\frac{4}{9} b w_s^2$ .

(5) Power due to beats between target side bands. This power is also one-half of that due to (1) of above, giving  $\frac{4}{9} b w_s^2$ .

(6) A sine wave due to beats between target carriers and associated side bands. Since these cross-products give cophasal voltages at the modulation frequency, they add linearly in voltage rather than in power. The voltage due to square-law detection of one target carrier with its side bands has amplitude  $2e_s^2$  or rms  $\sqrt{2e_s^2} = \sqrt{2} \frac{2w_s \delta}{3}$ . Since there are  $b/\delta$  target carriers the total rms voltage is  $\frac{2\sqrt{2}}{3} bw_s$ , with power  $\frac{8}{9} b^2 w_s^2$ .

114. The first five, of the above six, additional powers in the output of the postdetector band-pass filter have envelopes with probability density functions identical with that found with background noise alone. The sixth component is a sine wave, which, when added to the voltage due to background noise, will change the probability density functions as shown in Figure 39 (due to Rice, Ref. 8). If this additional component is considered only, we will err on the safe side since we neglect additional power which is present only when the target is present. The problem may then be solved by direct application of the work of Rice and Bennett. The largest of the five neglected powers is in a ratio of  $\frac{2}{3} \frac{w_s}{w_n}$  to the power found with background noise alone. For targets less than 10 db below background this error is less than 10 per cent.

115. The parameter  $a$  in Figure 39, which is the ratio between the amplitude of the sine wave to the rms noise, is then:

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$$a = \frac{\frac{4}{3} bw_s}{\sqrt{2bc} w_n} = \frac{2\sqrt{2}}{3} \sqrt{\frac{b}{c}} \frac{w_s}{w_n}. \quad (3)$$

116. From this equation, with the aid of Figure 40 (due to Bennett), the S curves of Figure 26 are calculated. For an example, consider the curve for  $r = 0$  and  $c = 1/10$ . The probability of commissive error has been chosen as 0.05, which means that the envelope of the output of the band pass filter must be less than the bias  $V$  with a probability of 0.95 when no signal is present (i.e.,  $a = 0$ ). Referring to Figure 40 we find  $V = 2.4$ . Let us assume  $a = 2$ ,  $b = 6,000$ . Then  $w_s/w_n = 0.00866$ , or -20.62 db, from equation (3) above. The curves in Figure 26 are plotted against peak signal-to-noise however. The peak intensity of a sine wave is  $8/3$  times its average intensity or 4.25 db larger. The peak signal-to-noise corresponding to  $a = 2$  is then  $-20.62 + 4.25 = -16.37$  db. Referring again to Figure 40 for  $a = 2$  and  $V = 2.4$  the probability that the bias is not exceeded when the signal is present is 0.59, which point is plotted in Figure 26.

117. When the frequency of the signal is known to within 10 cps there are 100 possible bands (for  $c = 1/10$  cps) which might exceed the bias when no signal is present. Therefore, the bias must be set higher to keep the total commissive error at 0.05. This accounts for the displacement to the right of the curve for  $r = 10$ ,  $c = 1/10$ .

118. The case for  $c = 1/2$  is complicated by the fact that five independent observations of filter outputs may be made in 10 seconds. This has been accounted for in two different ways. In Figure 26 the so called "simple trigger" criterion has been used, which means that the bias has been chosen so that if it is exceeded once in 10 seconds a signal is considered to be present.

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In Figures 41 and 42 the "majority report" criterion has been used, which means that three or more of the five observation must exceed the bias for a signal to be judged present. It is seen that the 1/2-cps system performs about 1 db better on the majority-report basis than on the simple-trigger basis. The difference between the 1/10-cps system and 1/2-cps system for  $r = 0$  is about 2 db and for  $r = 10$  cps is 1 db. This is not a great deal when it is considered that the 1/10-cps system requires five times as many filters as the 1/2-cps system when the frequency is not known. Even if analysis is done by frequency multiplication and a single filter it is more difficult to provide for 1/10-cps resolution than 1/2-cps resolution.

2. Listening Tests - Detection of Tones in Noise When the Signal Frequency is Unknown

119. In Figure 29a a comparison is made between the performance of the ear and that of an ideal detection system in the detection of tones of unknown frequency in noise. In order to determine the aural transition curve for tones of uncertain frequency, listening tests were run at these Laboratories. (Figure 28)

120. In most respects, the conduct of these tests followed the procedure described in Part I, Section C-1-d-(1) of this report. However, one new factor was introduced here. After each judgment the frequency of the signal was changed (within an uncertainty of 1200 cps). As in previous tests, three transmissions were made at each of six signal levels. In each of these three transmissions a different signal frequency was used, chosen from one of three bands (200-600, 650-1000, and 1050-1400 cps). These bands were chosen so that the critical bandwidth was approximately constant in each.

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121. The transition curves were considered for each frequency group separately. In Figure 28, the results for the 650- to 1000-cps range are shown. Tests were also run at a fixed frequency of 900 cps in order to determine the threshold shift when the signal frequency is not known in advance. Zero db S/N on the abscissa represents equal signal-to-noise in a critical band at 900 cps.

3. Detection of a Tone in Noise by N Observations of a Single Filter  
rcps Wide

122. The calculations and the assumptions made in finding the lower set of curves in Figure 30 are discussed in this section. In order that the curves given by Rice and Bennett (Ref. 10) for the envelopes of a sine wave added to noise may be used it is necessary that the bandwidth of the noise be small compared to the center frequency of the band, and that the frequency of the tone be in the center of the noise band. The first condition may be satisfied by considering the band of interest to be heterodyned to a high frequency. The second condition is not satisfied in this problem because the frequency of the tone is assumed to fall anywhere in the noise band with equal probability. This discrepancy has been ignored, the assumption being made that the effect is small compared to the difference between the two systems being compared.

123. With these assumptions the problem reduces to the computation of the probability of emissive error for a given probability of commissive error. Decision as to the presence or absence of a signal is made on the basis of a majority of the observations of the envelope of the output of the single filter.

124. Let:

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$P_o$  = probability of being right on one observation  
when signal is present

$Q_o$  = probability of being wrong on one observation  
when signal is present

$R_o$  = probability of being right on N observations -  
taking majority report - when signal is present

$P_c$ ,  $Q_c$ ,  $R_c$ , are defined as  $P_o$ ,  $Q_o$ ,  $R_o$ , for the case of  
signal not being present.

125. In the sequel, equations containing the above defined symbols  
are written without subscripts. These equations are correct if for  $(P, Q, R,)$   
one substitutes either the set  $(P_c, Q_c, R_c,)$  or the set  $(P_o, Q_o, R_o,)$ . Thus  
the equation,

$$P + Q = 1 \quad (1)$$

means,

$$\begin{aligned} P_c + Q_c &= 1 \\ P_o + Q_o &= 1 \end{aligned} \quad (2)$$

126. Let  $x$  be the number of times that one is right in  $N$  observations  
of either noise or signal plus noise. If  $N$  is large,

$$R = \sum_{x=0}^N \binom{N}{x} P^x Q^{N-x} \quad (3)$$

$$\frac{N}{2} \leq x \leq N$$

where,  $\binom{N}{x} = \frac{N!}{x!(N-x)!}$

is the number of combinations of  $N$  observations -  $x$  correct and  $(N-x)$  incorrect.  
The probability of exactly  $x$  being correct in  $N$  trials is  $\binom{N}{x} P^x Q^{N-x}$ .  $R$  is the  
probability that  $x$  is greater than  $N/2$ , which is the sum of the probabilities  
that  $x$  equals  $N$ ,  $x$  equals  $N-1$ , .. all the way down to  $x$  equals  $\frac{N+1}{2}$  or  $x$  equals

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$\frac{N+2}{2}$  depending on whether N is odd or even.

127. From page two hundred of Cramer's "Mathematical Methods of Statistics", it follows that:

$$R = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \sqrt{\frac{N}{2}} \frac{P-Q}{2\sqrt{PQ}} \right) \right] \quad (4)$$

where,

$$\operatorname{erf} y = \frac{2}{\sqrt{\pi}} \int_0^y e^{-s^2} ds. \quad (5)$$

128. It follows from equations (1) and (4) that\*

$$2Q = 1 \pm \sqrt{1 - \frac{1}{1 + \frac{2}{N} [\operatorname{erf}^{-1}(2R-1)]^2}} \quad (6)$$

where,

$$W = \operatorname{erf} y \text{ implies } y = \operatorname{erf}^{-1} W, \text{ and conversely.} \quad (7)$$

129. Let it be assumed that the commissive error on N pulses is five per cent, or, equivalently,  $R_c = .95$ . From equation (6) one may find  $Q_c$  which is a function of N.

$$Q_c = Q_c(N) \quad (8)$$

130. Tables exist for

$$\left\{ \begin{array}{l} 1 - Q_0(\alpha, \beta) = \int_{\beta}^{\infty} ve^{-\frac{v^2 + \alpha^2}{2}} I_0(\alpha v) dv \\ Q_c(\beta) = \int_{\beta}^{\infty} ve^{-v^2/2} dv = e^{-\beta^2/2} \end{array} \right\} \quad (9)$$

Now,  $Q_c$  is known. From the last equation, one easily determines  $\beta$  as a function of N;  $\beta$  is the bias level.

$$\beta = \beta(N) \quad (10)$$

---


$$2Q = 1 - \sqrt{\quad} \quad \text{if } R > \frac{1}{2} \quad 2Q = 1 + \sqrt{\quad} \quad \text{if } R < \frac{1}{2}$$

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131. The signal-to-noise ratio is  $\alpha$ . Various values of  $\alpha$  are assumed;  $\beta(N)$  is known. From the first of equations (9), one easily determines  $Q_0(\alpha, \beta)$  where,

$$Q_0(\alpha, \beta) = Q_0(\alpha, \beta(N)) = Q_0(\alpha, N). \quad (11)$$

Thus,  $Q_0$  is a function of signal-to-noise ratio and the number of pulses.

Knowing  $Q_0$ , from equations (1) and (4) one determines  $R_0$  where,

$$R_0 = R_0(\alpha, \beta(N)) = R_0(\alpha, N). \quad (12)$$

132. For various  $N$ ,  $R_0(\alpha, N)$  has been plotted versus  $10 \log_{10} \frac{\alpha^2}{2}$ . Thus, given the commissive error on  $N$  pulses is .05, one minus the omissive error on  $N$  pulses has been plotted versus the signal-to-noise ratio in decibels.

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SECTION B - REFERENCES

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SECTION C - ILLUSTRATIONS

<u>Fig. No.</u>	<u>Title</u>
1	Signal-to-Noise Improvement Due to Comb Filter (FTL 14070)
2	Frequency Response of Comb Filter (FTL 17123)
3	Method of Realizing Comb Filter
4	Performance of Comb Filter in Terms of Gain
5	Copper Tube for Acoustic Delay Line (FTL 15670)
6	Acoustic Delay Line - Top View (FTL 17150)
7	Frequency Response of Acoustic Delay Line
8	Comb-Filter Circuit (B-2070296)
9	Block Diagram of Comb Filter and Modulator (IL 13422-1)
10	Idealized Action of Spectrum-Translating Modulator (IL 13261-1)
11	Band-Pass Filter (A-2070289)
12	Operation of Variable-Frequency Oscillator
13	Volt-Ampere Characteristic of Thyrite
14A	Circuit of Oscillator and Modulator (B-2070287)
14B	Input Circuit of Comb Filter (B-2070298)
15	Delay Line and Power Supply - Top View (FTL 17799)
16	Comb Filter - Side View (FTL 17794)
17	Comb Filter - Bottom View (FTL 17800)
18	Plug-In Units of Comb Filter (FTL 17795)
19	Input Chassis of Comb Filter (FTL 17805)
20	Feedback Amplifier of Comb Filter (FTL 17803)
21	Oscillator and Output Chassis of Comb Filter (FTL 17804)

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<u>Fig. No.</u>	<u>Title</u>
22	Apparatus for Comb-Filter Listening Test
23	Improvement Due to Comb Filter (1-KC Tone - New Model)
24	Use of Comb Filter in Detection of Tones from USS TUSK (FTL 18072)
25	Detection of Tones in Self-Noise of USS HALFBEAK, While Hovering (FTL 18762)
26	Detection of an Amplitude-Modulated White Noise in A White-Noise Background
27	Effect of Uncertainty of Signal Frequency in Detection of Tones in White Noise
28	Aural Detection of Tones in White Noise - Effect of Uncertainty in Signal Frequency
29A	Comparison of Aural Detection of Tones in Noise with Ideal Detection Systems
29B	Improvement Due to Majority Report of H Observations
30	Detection of a Tone in a Band of Noise - Ideal System Compared with Single Filter with Many Observations
31	Detection of Amplitude-Modulated White Noise (100%) in a White-Noise Background
32	Build-up of a Tone in a Frequency-Scanning Analyzer
33	Detection of a Tone in Noise by a Frequency-Scanning Analyzer BW = 15 CPS
34	Detection of a Tone in Noise by a Frequency-Scanning Analyzer BW = 36 CPS
35	Detection of a Tone in Noise by a Frequency-Scanning Analyzer BW = 21 CPS
36A	Facsimile Recorder, Stylus-Drive Mechanism (FTL 18765)
36B	Facsimile Recorder, Paper-Feed Mechanism (FTL 18763)
37A	Response of Facsimile Recorder to Tones and Noise (FTL 19112)
37B	Response of Facsimile Recorder, Detection of Pulsed Tone in Noise (FTL 19111)

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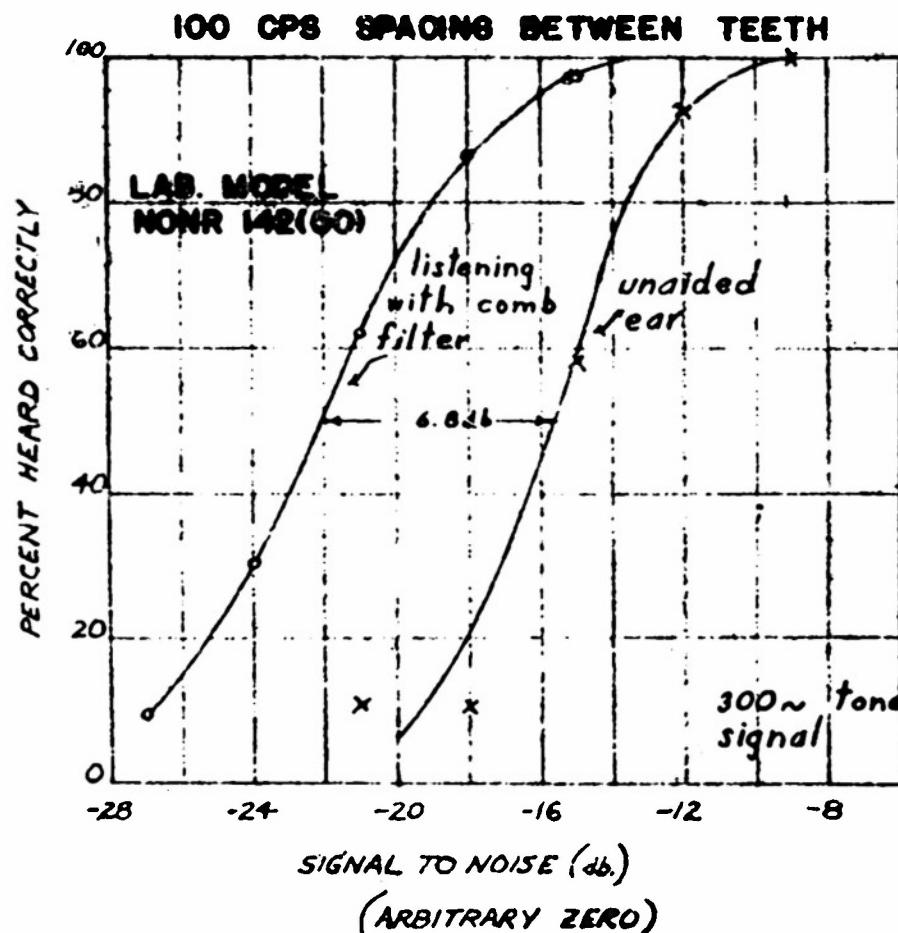
<u>Fig. No.</u>	<u>Title</u>
38	Oscillograms of Idealized Background and Target Noises
39	Probability Density of Envelope R of $I(t) = P \cos pt + I_n$ (FTL 16439)
40	Distribution Function of Envelope R of $I(t) = P \cos pt + I_n$ (FTL 16440)
41	Detection of an Amplitude-Modulated White Noise in a White-Noise Background (Comparison of 1/10-CPS Filter with 1/2-CPS Filter - Majority Report Criterion)
42	Detection of an Amplitude-Modulated White Noise in White-Noise Background (Effect of Choice of Majority Report or Simple Trigger Criterion)

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## Signal to Noise Improvement Due to Comb Filter



5 observers  
3 tests on  
each system  
11/24/50

*Figure 1*

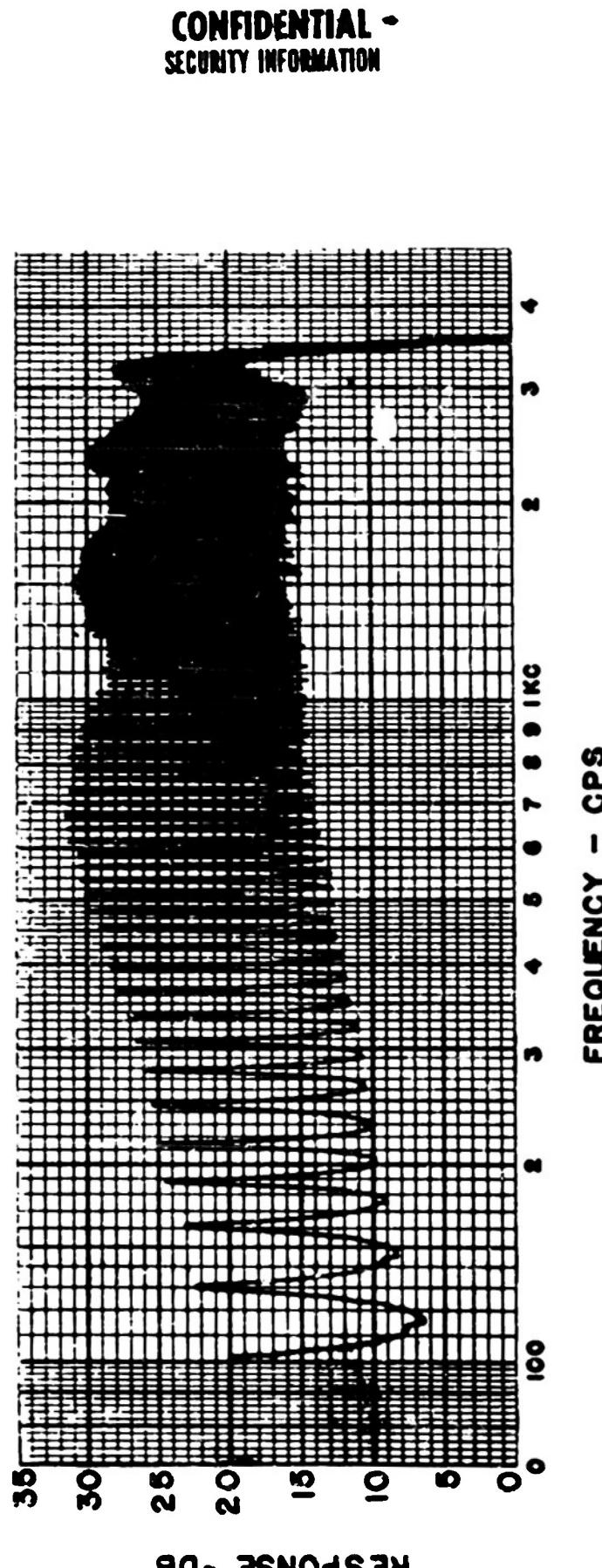
filter bw @ 300 $\omega$   
of 4~.

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FTL 14070

COMB FILTER  
FREQUENCY RESPONSE

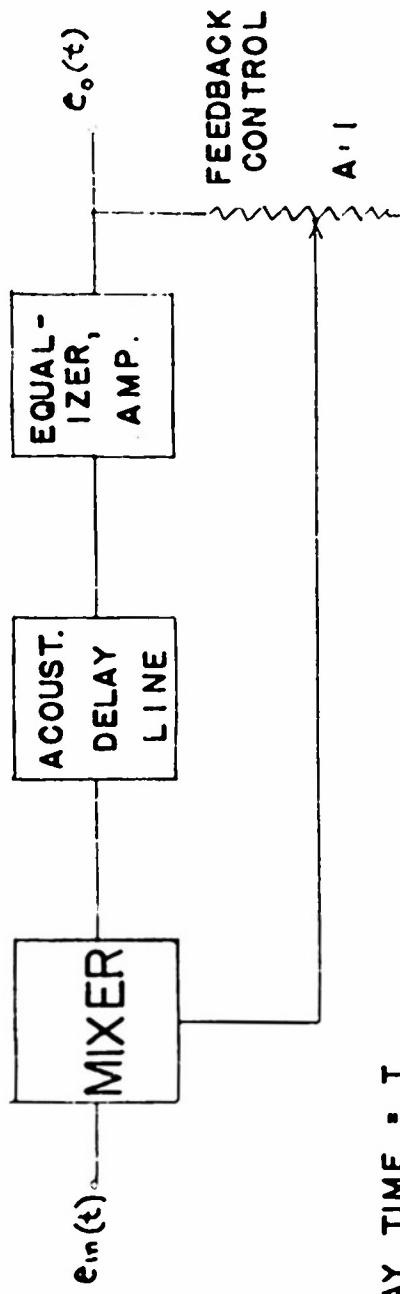


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FTL 17123

FIGURE 2

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DELAY TIME = T  
OPEN LOOP GAIN = K

$$\text{TRANSFER FUNCTION } |Y_p| = \frac{1}{[1 + (KA)^2 - 2KA \cos \omega T]^{1/2}}$$

$$60\text{B BANDWIDTH} = \frac{\sqrt{3}}{\pi T} \frac{1-AK}{\sqrt{AK}}$$

PEAK : VALLEY RATIO =  $\frac{1+AK}{1-AK}$  BOTH RELATIONS ARE PLOTTED IN FIG 4

### METHOD OF REALIZING COMB FILTER

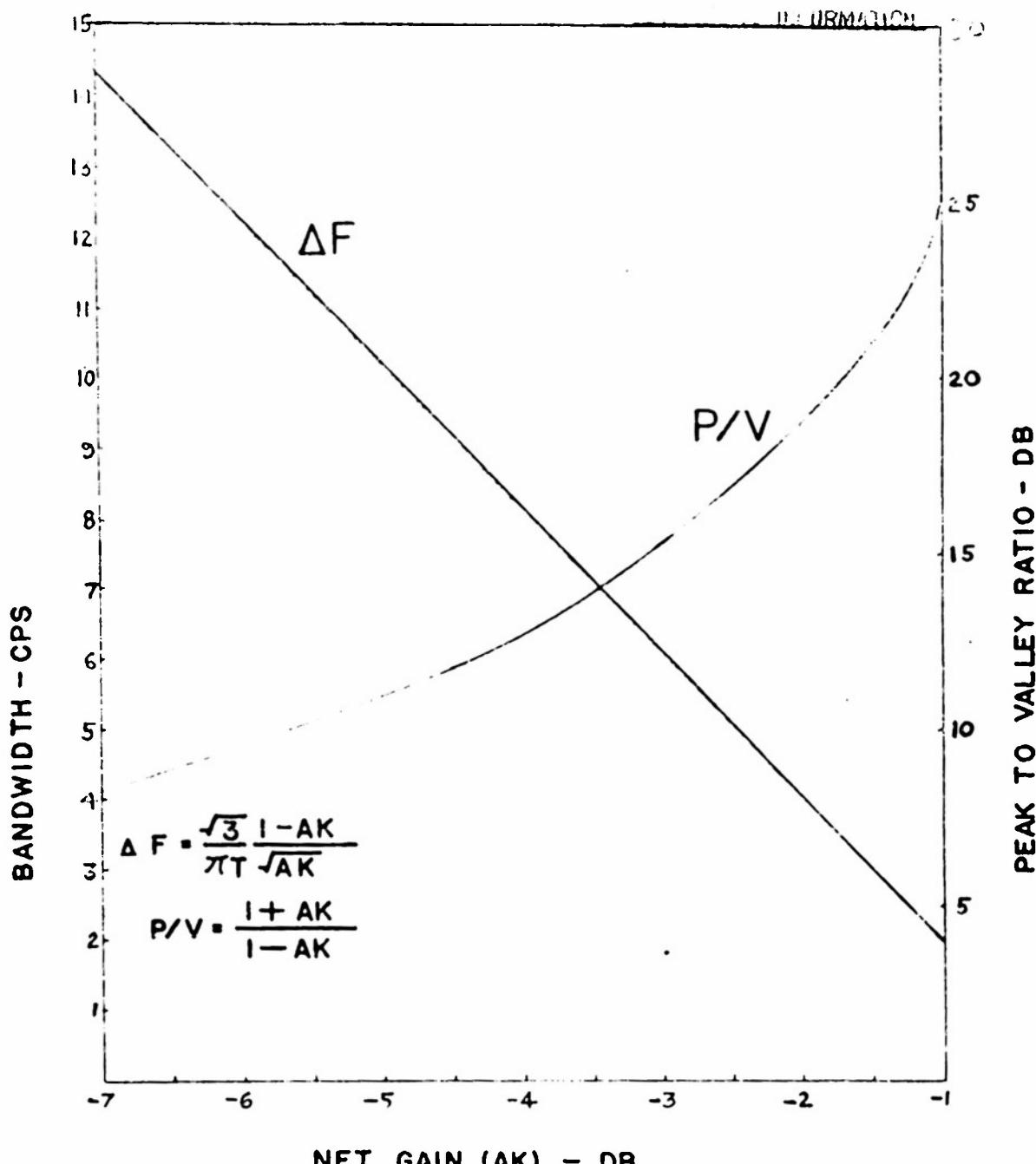
FIGURE 3

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TOLERANCE UP TO AND ABOVE 8 8 TO 84 84	MATERIAL	TITLE			
DEC. DIM. $\pm .00$ $\pm .010$ $\pm .018$	FINISH	ISSUED	USED WITH	APPD	DWN.
FRACT. DIM. $\pm \frac{1}{64}$ $\pm \frac{1}{32}$ $\pm \frac{1}{16}$ UNLESS OTHERWISE SPECIFIED					
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**FIGURE 4**

## PERFORMANCE OF COMB FILTER IN TERMS OF GAIN

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TOLERANCE UP TO ABOVE ABOVE 8 8 TO 84 84	MATERIAL	TITLE	
		ISSUED	USED WITH
DEC. DIM.	FINISH	APPD	OWN.
$\pm .005$	$\pm .010$		
$\pm .04$	$\pm .08$		
UNLESS OTHERWISE SPECIFIED			

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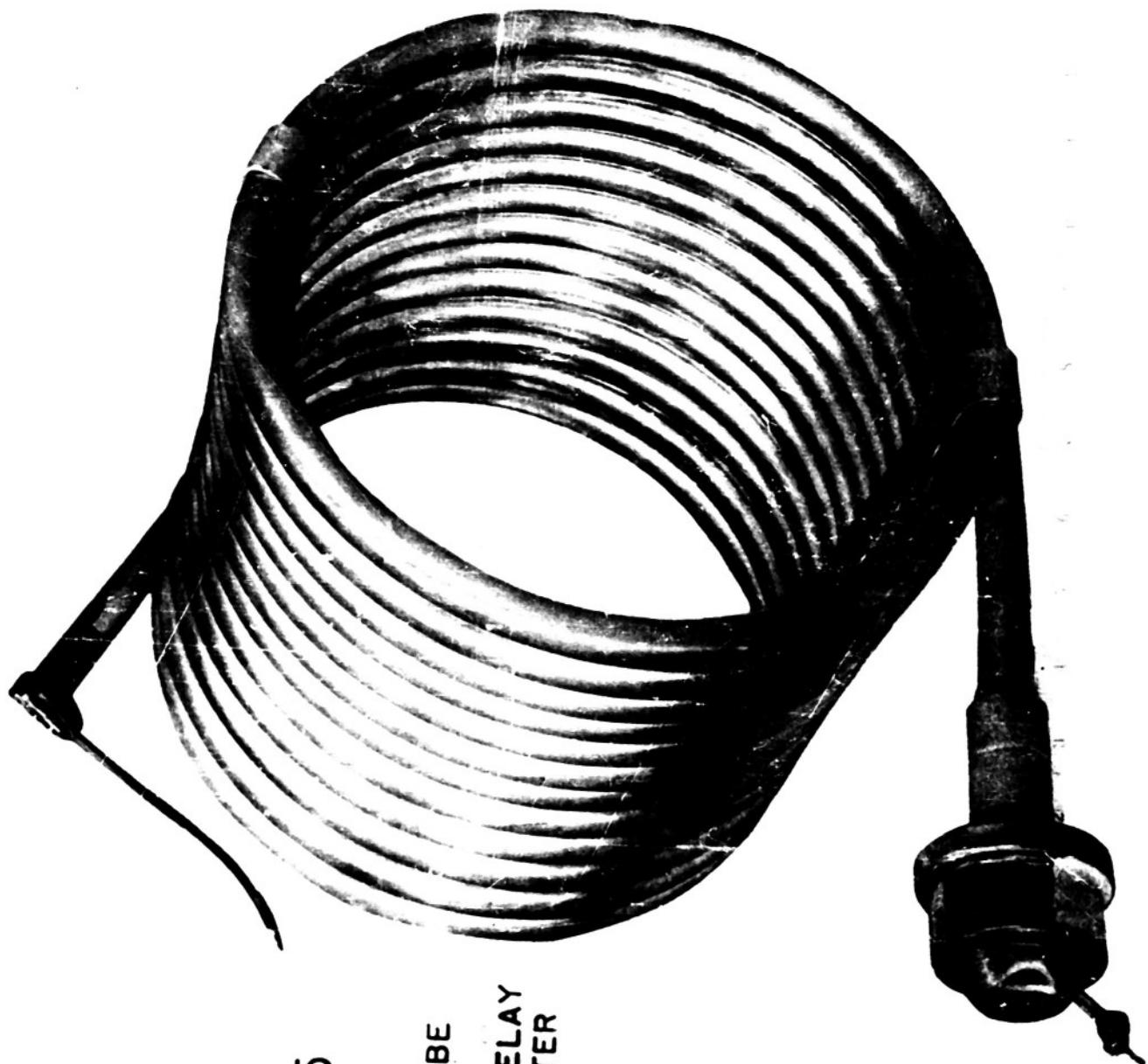


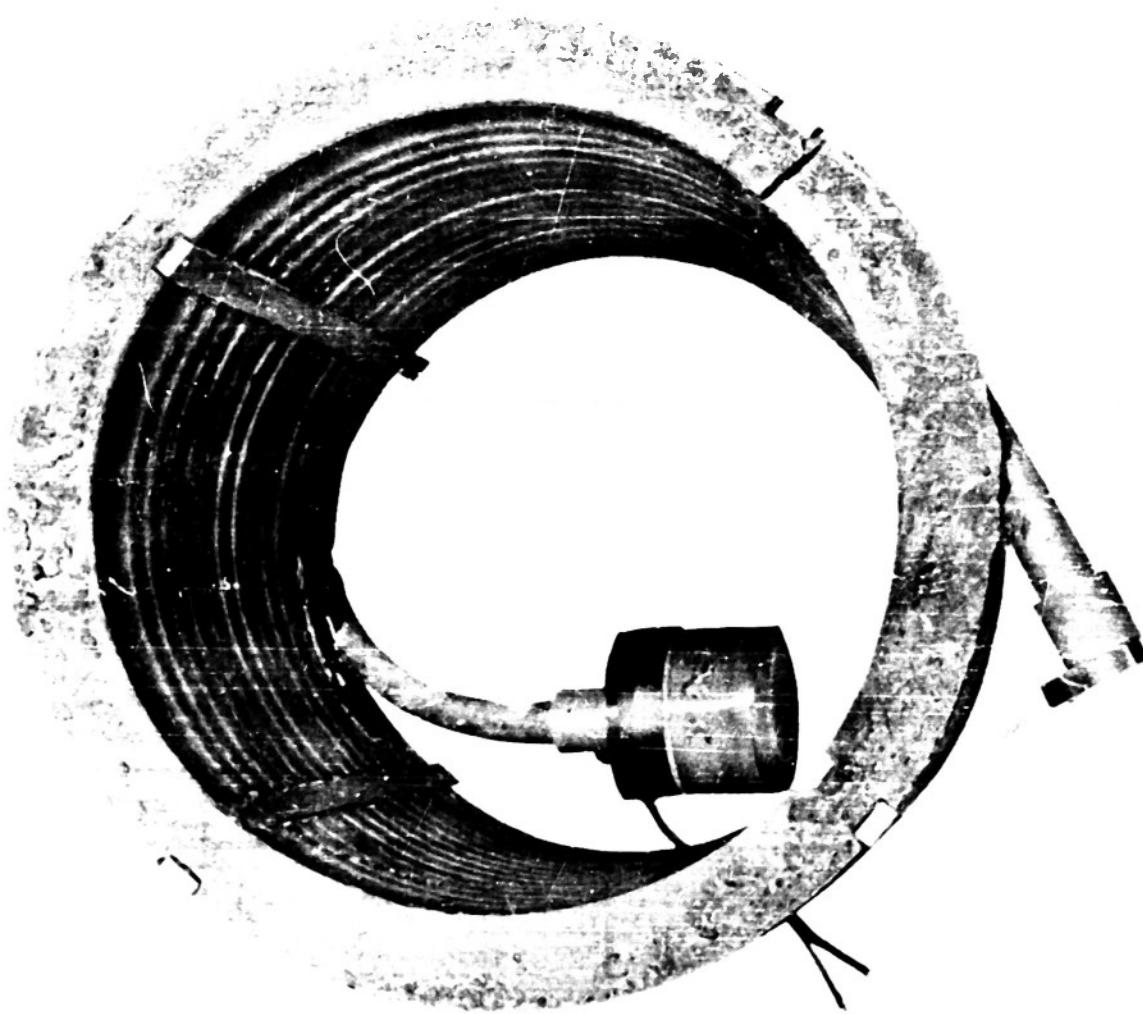
FIG. 5

COPPER TUBE  
FOR  
ACOUSTIC DELAY  
COMB FILTER

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FTL 13620

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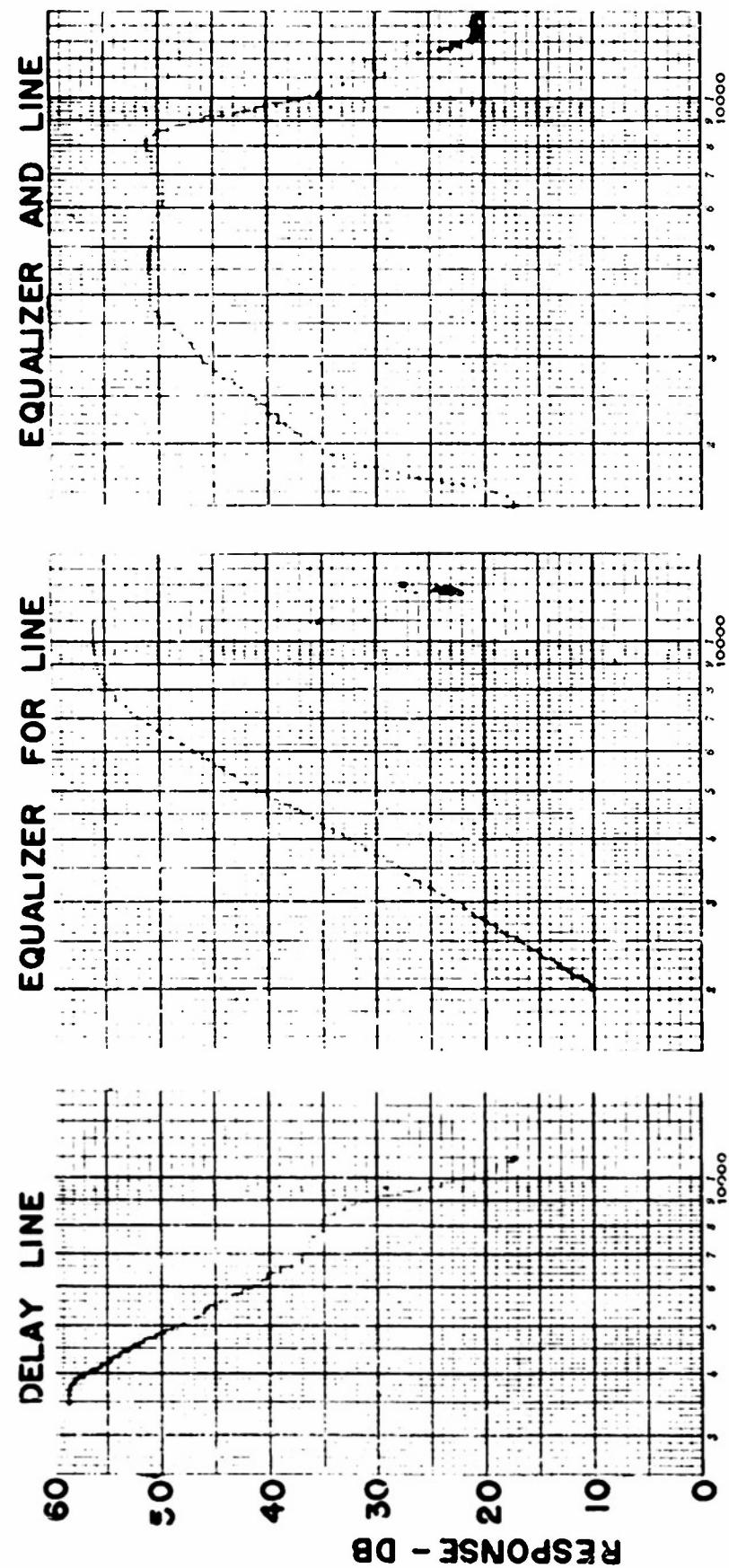


ACOUSTIC DELAY LINE  
TOP VIEW  
FIG. 6

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PTC 12-35

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FREQUENCY RESPONSE OF ACOUSTIC DELAY LINE

FIGURE 7

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B-2070296  
DRAWING NO.

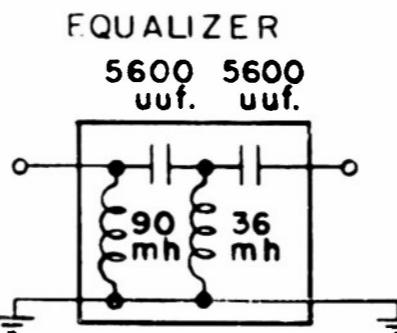
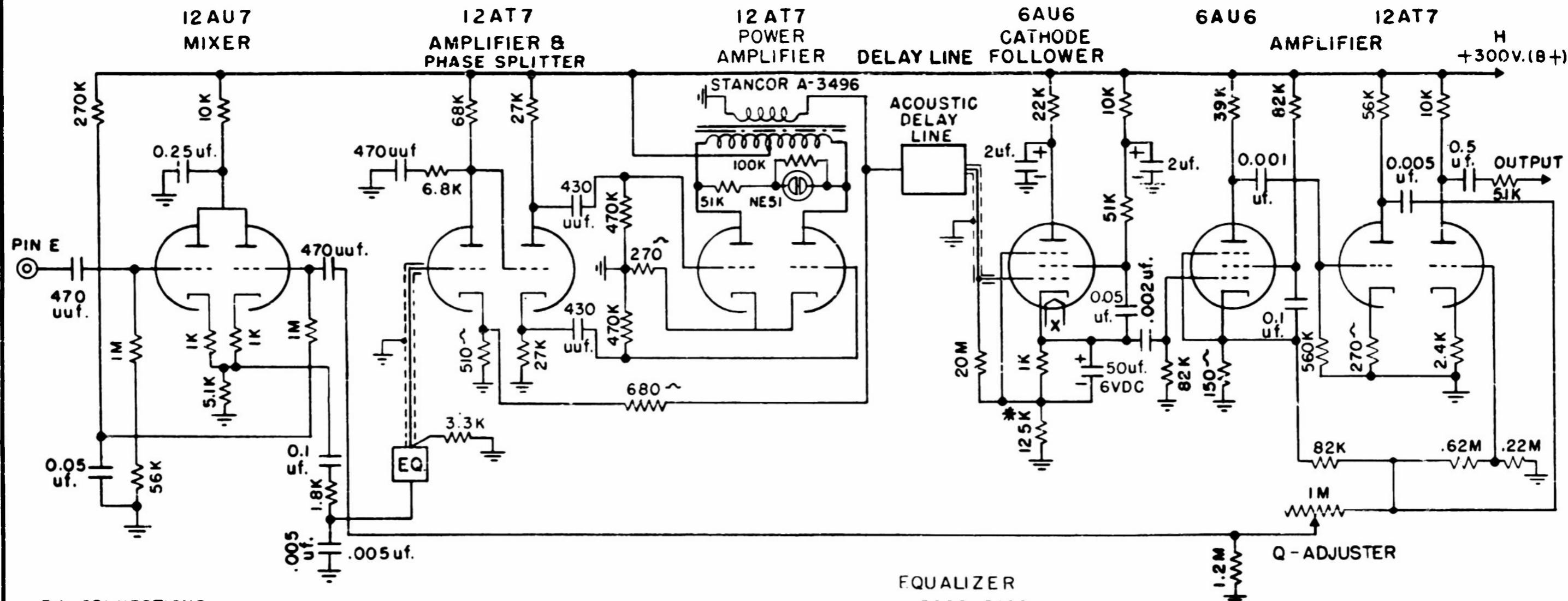
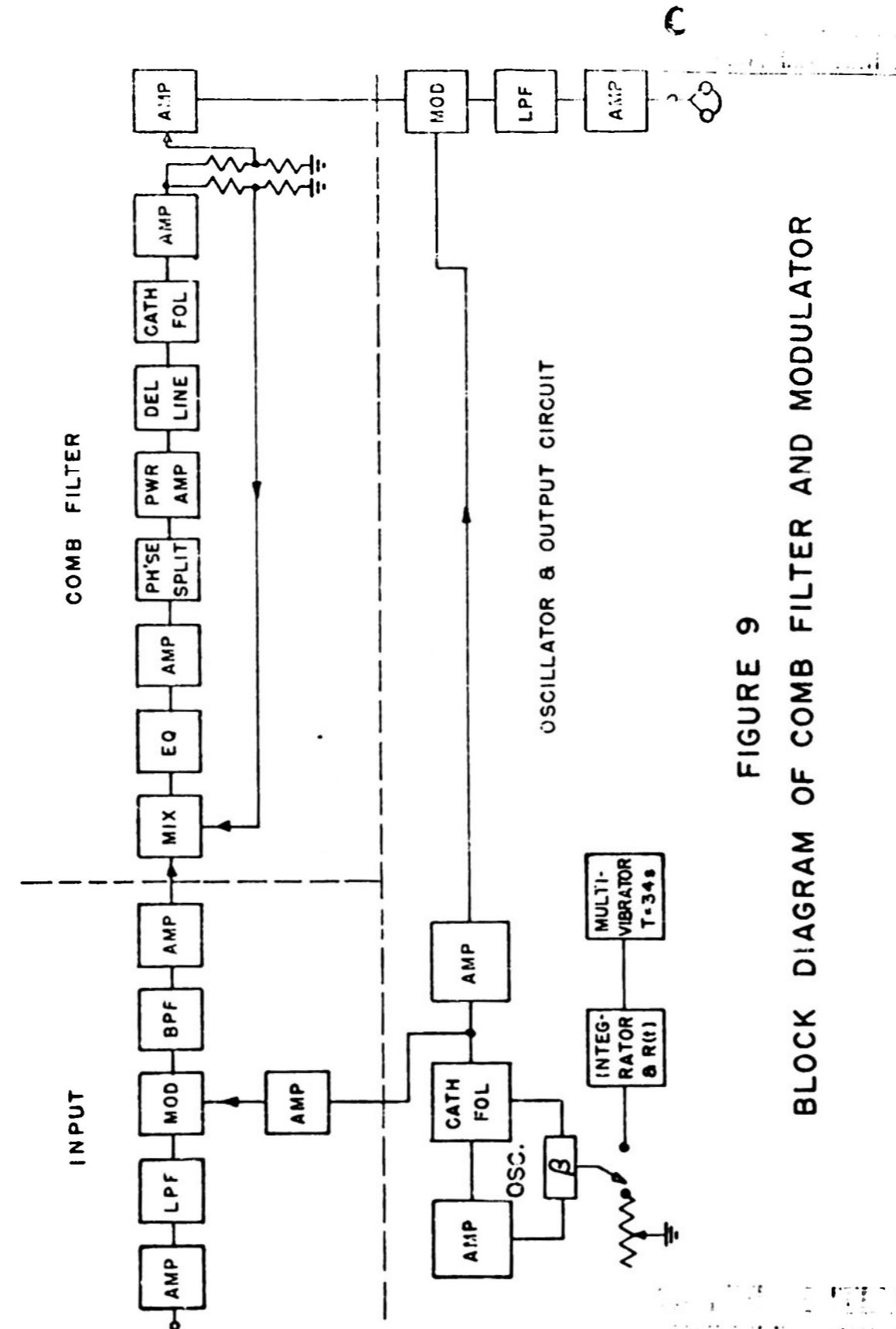


FIGURE 8

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<b>ORIGINAL ISSUE</b>	<b>REVISIONS</b>	UNLESS OTHERWISE SPECIFIED			FIRST USED ON			Federal Telecommunication Laboratories, Inc. NUTLEY, N. J. U. S. A.		
		ALL DIMENSIONS IN INCHES								
TOLERANCES						<b>NEXT ASSEMBLY</b>	<b>BILL OF MATERIAL</b>			
BASIC DIM.	FRACTIONS	DECIMALS								
UNDER 0	$\pm \frac{1}{64}$	$\pm .008$								
0 TO 24 INCL	$\pm \frac{1}{32}$	$\pm .010$								
OVER 24	$\pm \frac{1}{16}$	$\pm .015$								
ANGLES $\pm$	ECCENTRICITY TIL.		<b>SCALE</b>	<b>DES.</b>	<b>ENG.</b>	<b>MATERIAL</b>				
HOLE DIA. $\pm$	SURFACES ✓									
COMMERCIAL TOLERANCES APPLY TO STOCK SIZES						DRAWN	FAS	E OF M	DATE	
						CHG.			8-14-52	
						APPRO.			B	
									2070296	



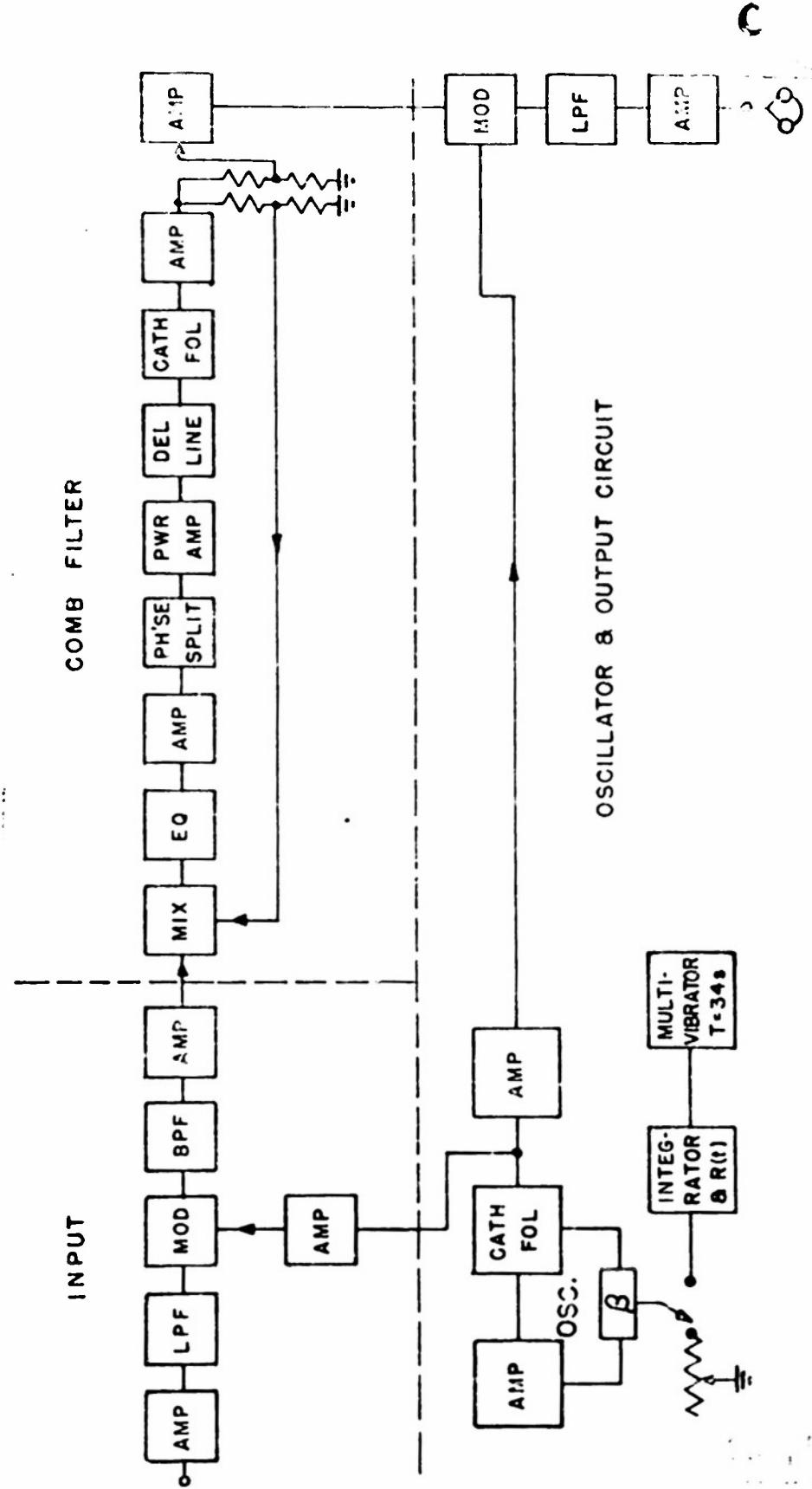
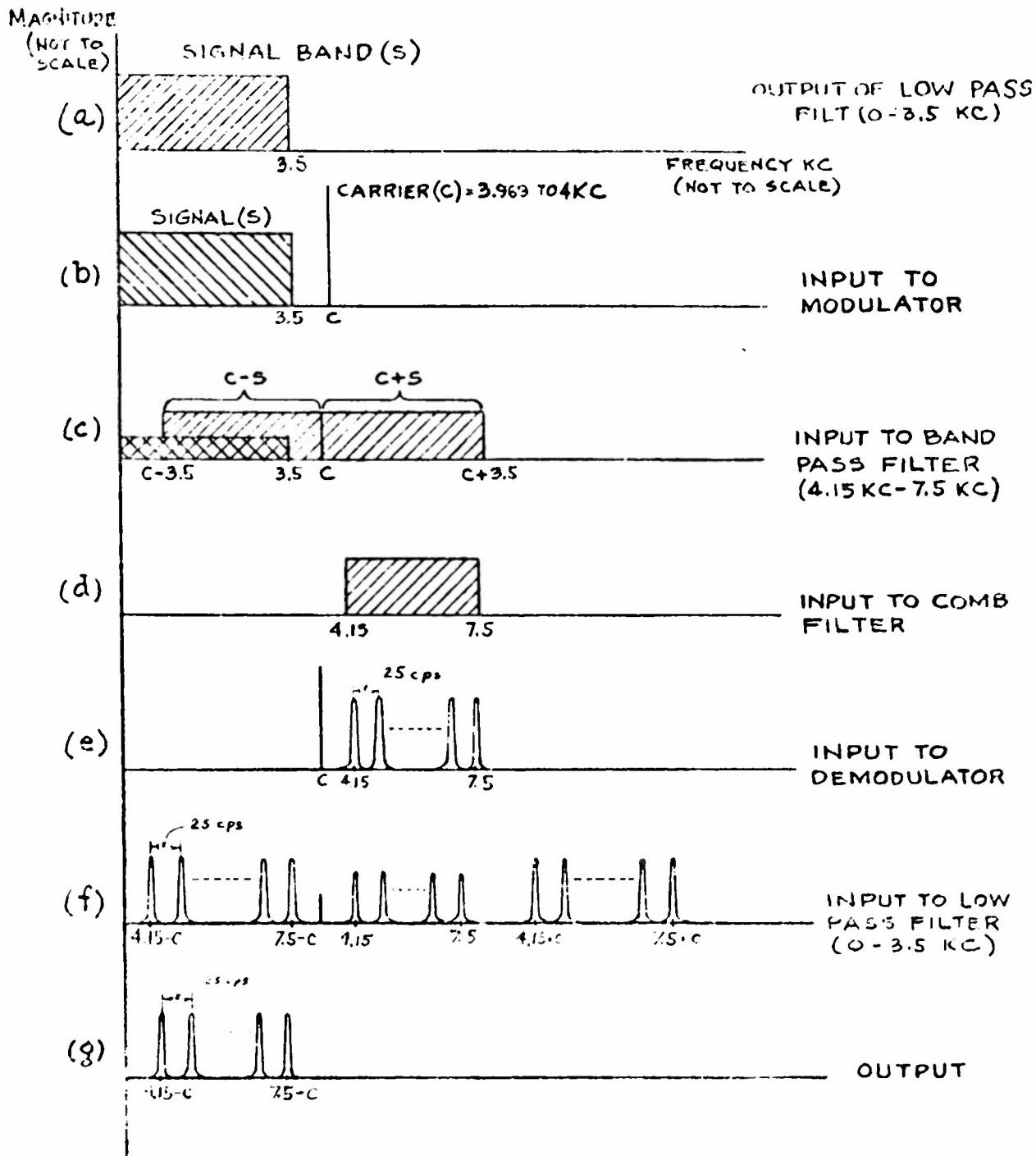


FIGURE 9

BLOCK DIAGRAM OF COMB FILTER AND MODULATOR

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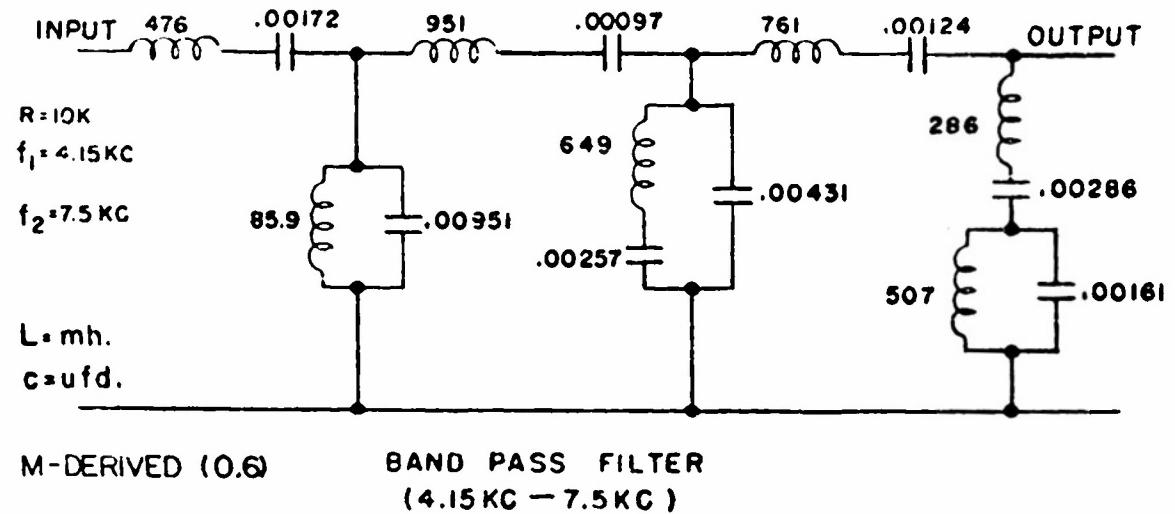
IDEALIZED ACTION OF SPECTRUM

TRANSLATING MODULATOR

FIG. 10

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REF ID: A62611

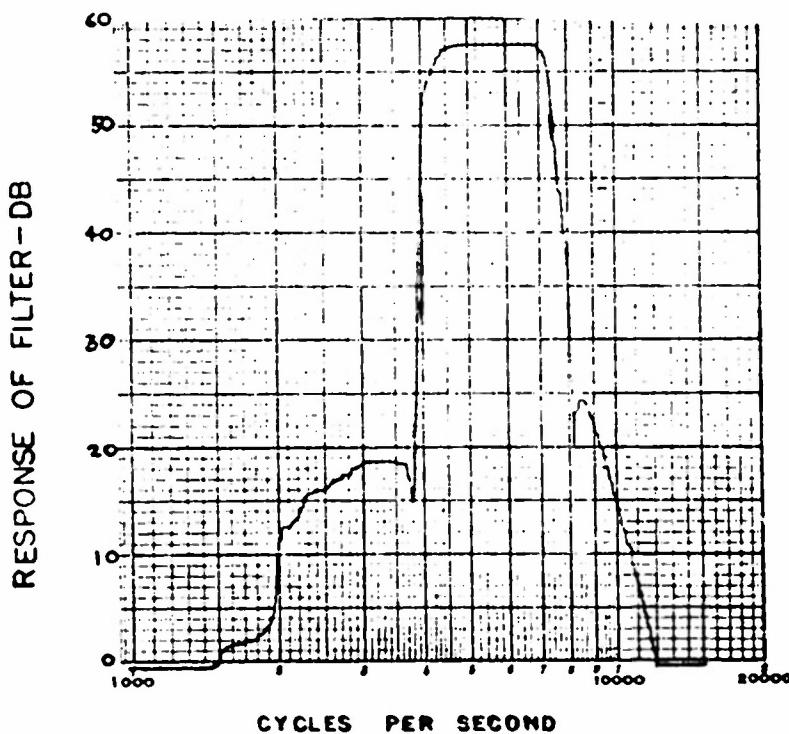
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FREQUENCY  
RESPONSE  
OF  
FILTER

FIGURE 11

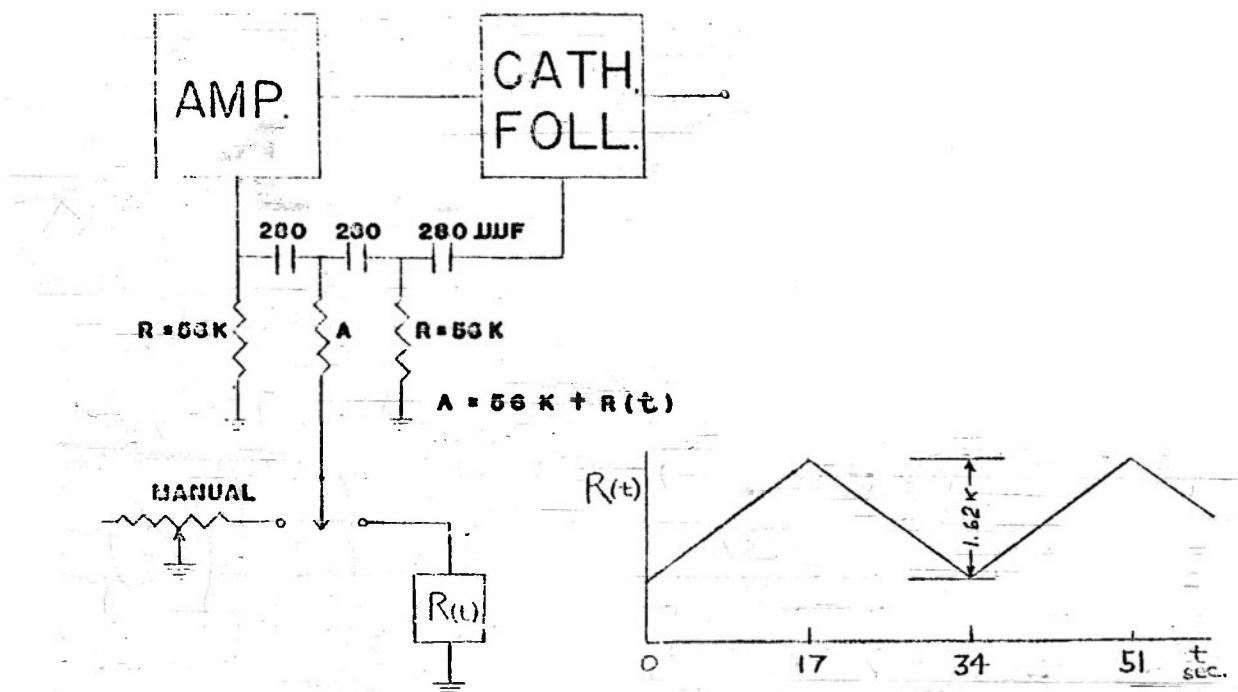
FIG. NO. 114(8)



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A-2070289-A

## VARIABLE FREQUENCY OSCILLATOR



$$F_0 = \frac{1}{2\pi GRC}$$

FOR SMALL FREQUENCY CHANGES,  
 $A \approx R$ , AND  $\frac{\partial A}{\partial F}$  REDUCES TO  $-\frac{3A}{F_0}$

$$\frac{F}{F_0} = \sqrt{\frac{3}{1 + 2A/R}}$$

$$\frac{\partial A}{\partial F} = -\frac{(2A + R)}{F}$$

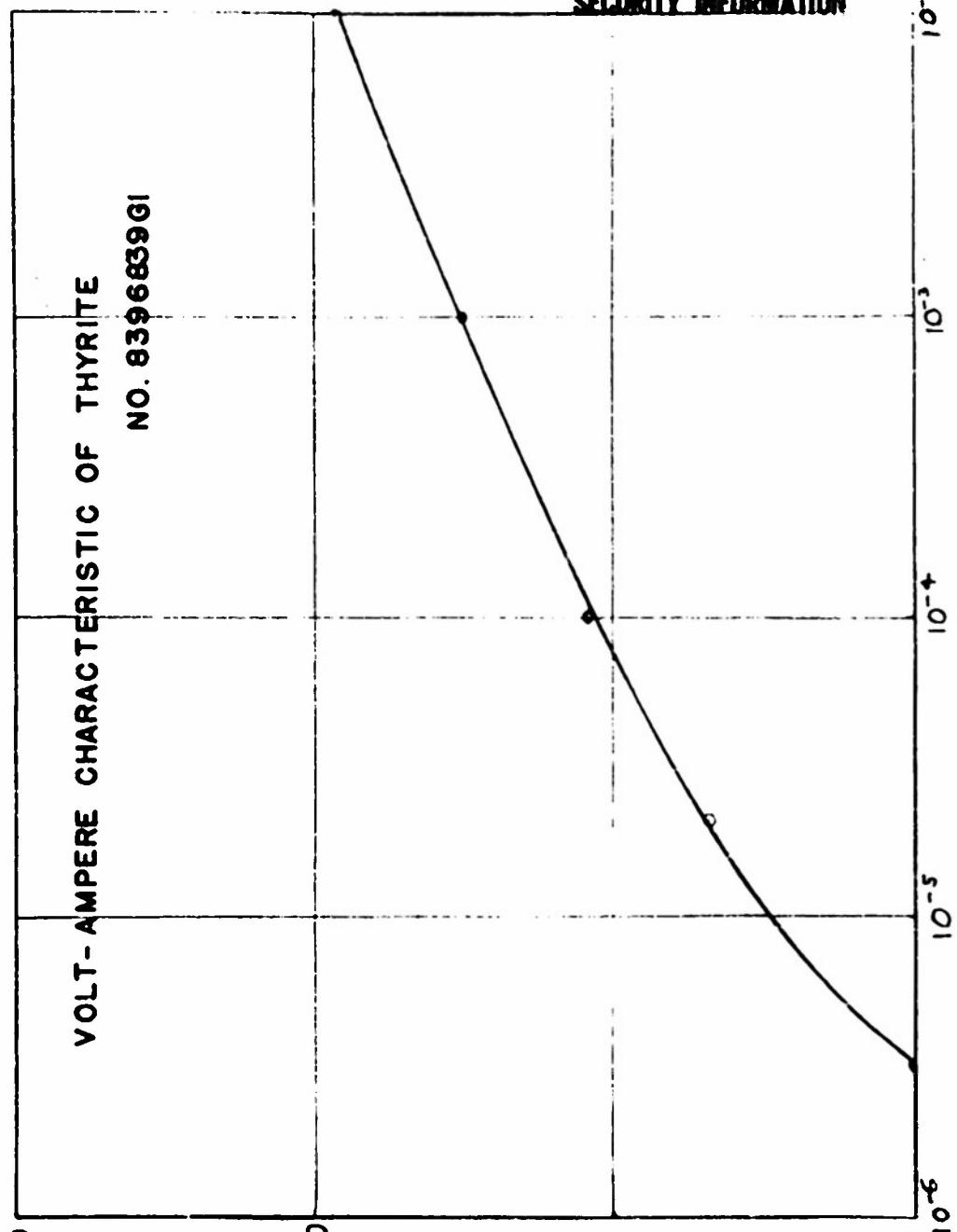
### OPERATION OF FREQUENCY SHIFTER

FIGURE 12

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TOLERANCE UP TO .005 .010 .015 ABOVE .005 .010 .015	MATERIAL	TITLE			
DEC. DIM. FRAC. DIM. UNLESS OTHERWISE SPECIFIED	FINISH	ISSUED	USED WITH	APPD	DWN.

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INSTANTANEOUS CURRENT - AMPS

FIGURE 13

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TOLERANCE	MATERIAL	TITLE			
UP TO ABOVE ABOVE		ISSUED	USED WITH	APPD	DWN.
0 .010 .015					
DEC. DIM. ± .005 ± .010 ± .015					
FRACT. DIM. ± $\frac{1}{64}$ ± $\frac{1}{32}$ ± $\frac{1}{16}$	FINISH				
UNLESS OTHERWISE SPECIFIED					
Federal Telecommunication Laboratories, Inc.				-1	

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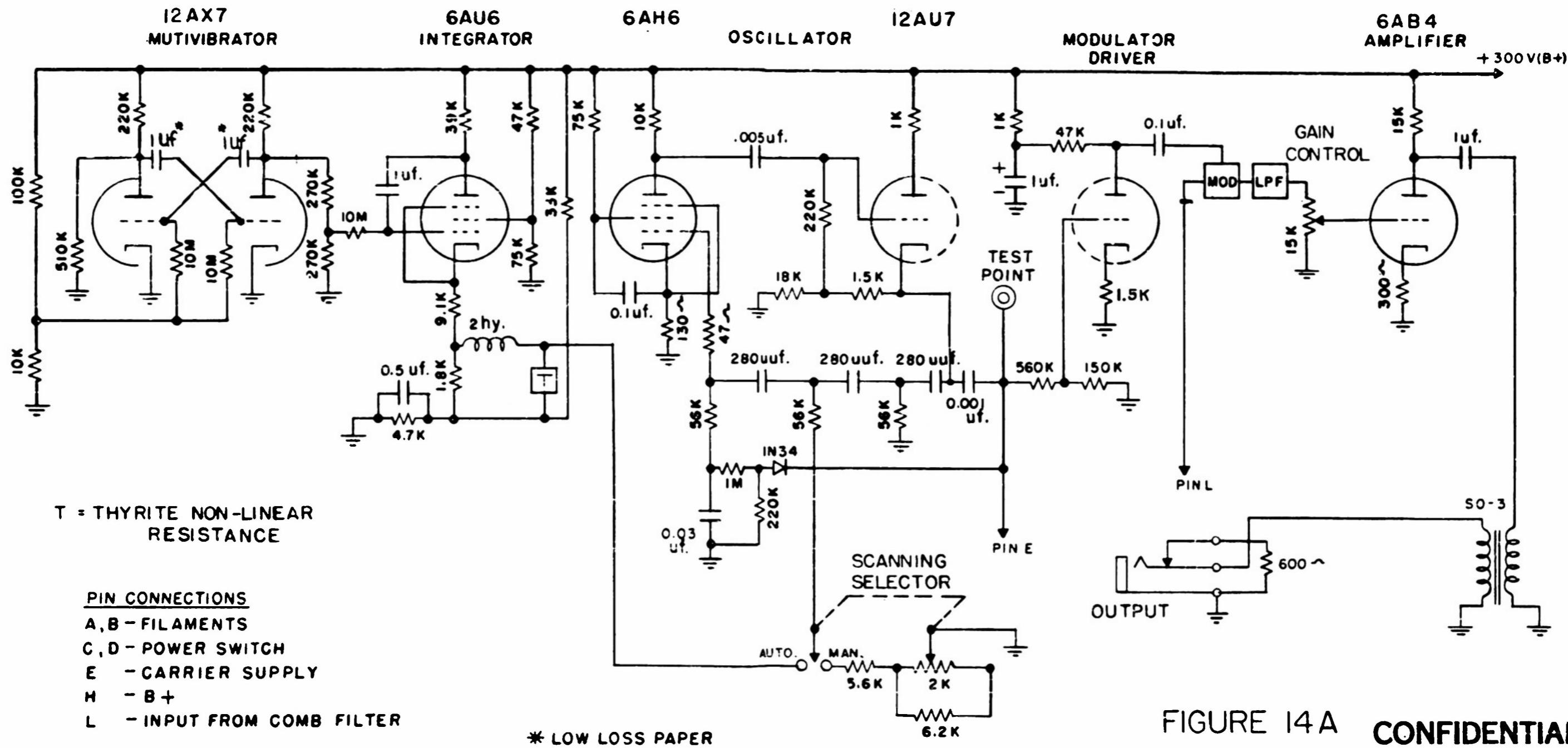
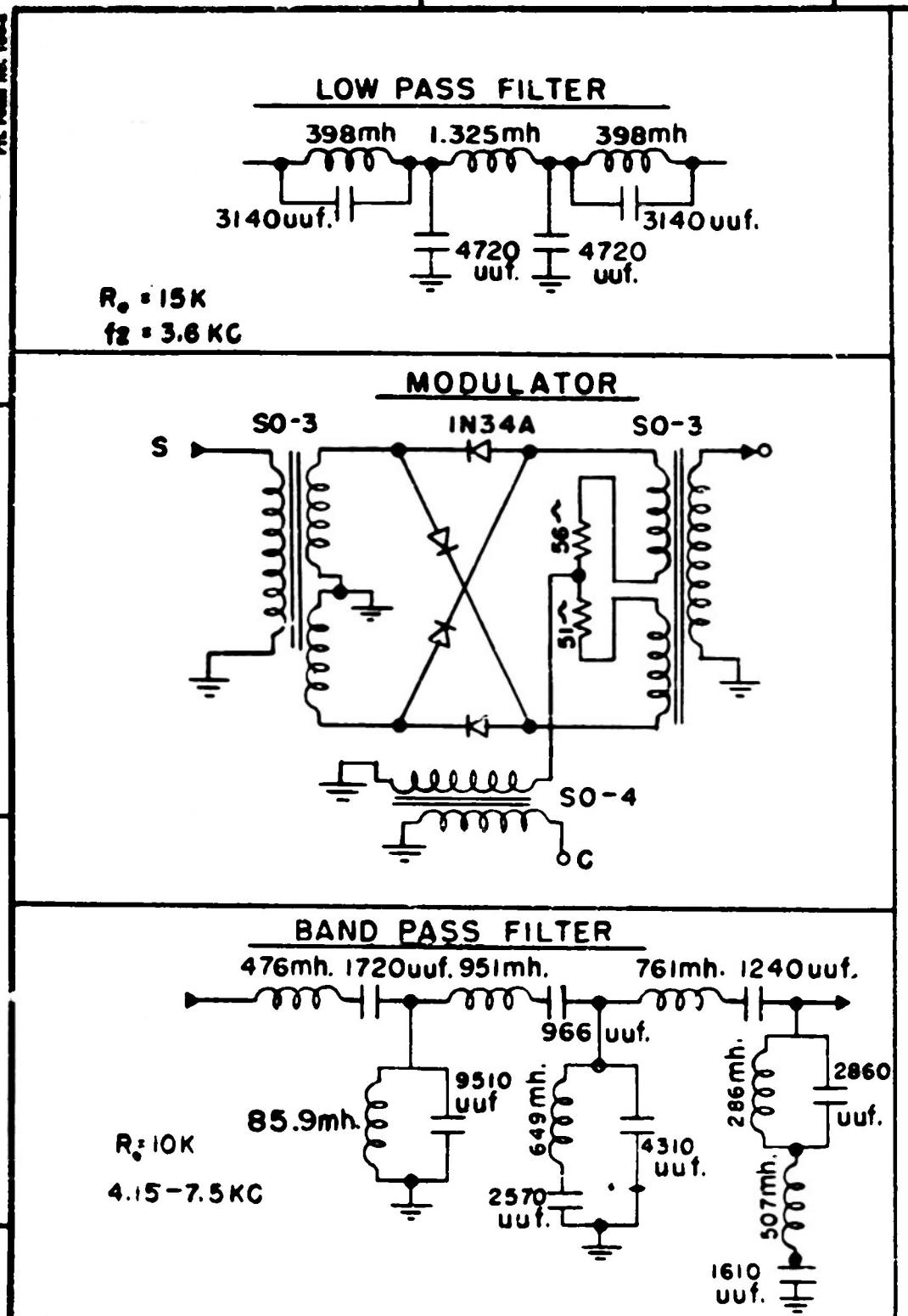


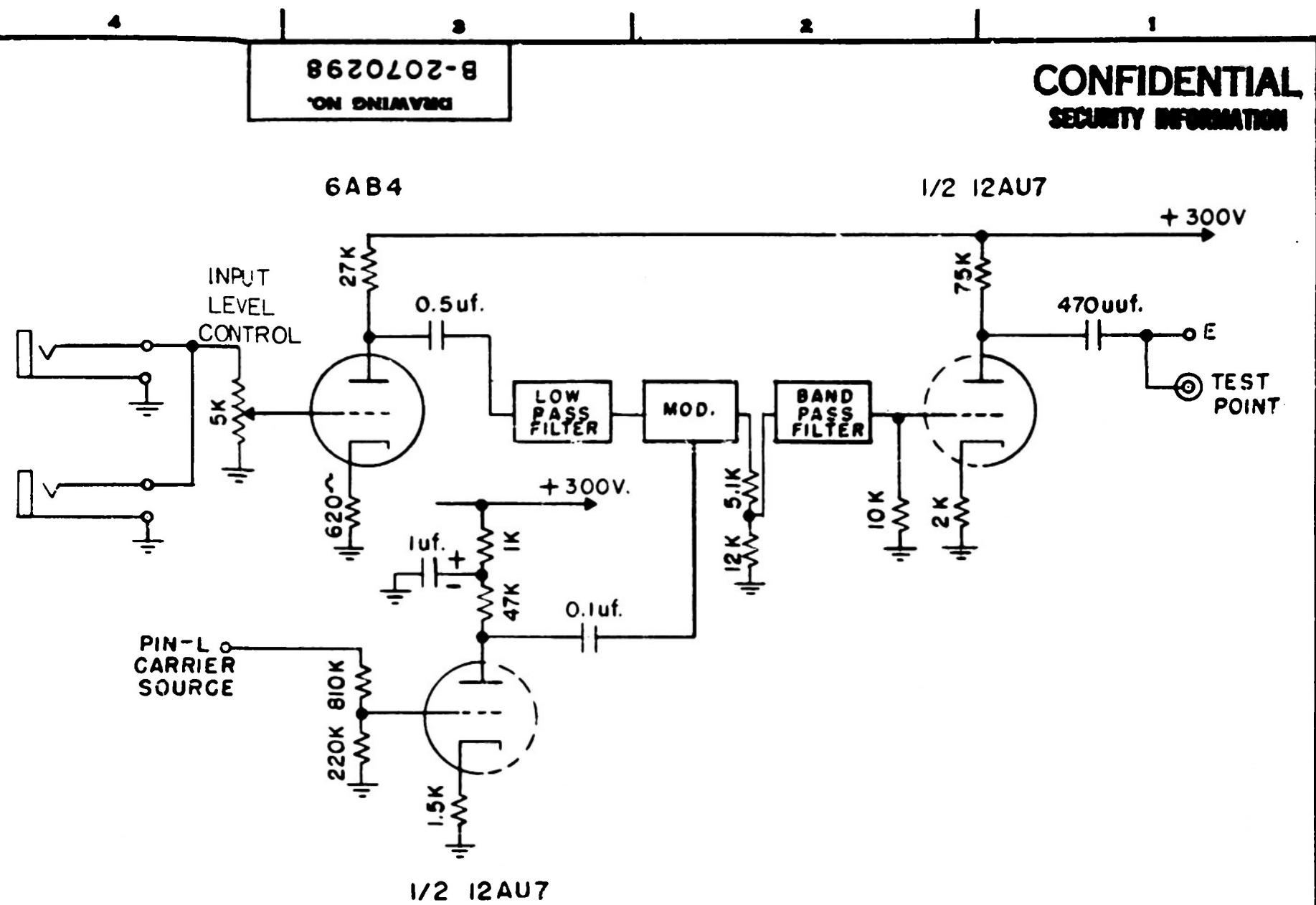
FIGURE 14A **CONFIDENTIAL**  
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A ORIGINAL ISSUE									Federal Telecommunication Laboratories, Inc. NUTLEY, N. J., U. S. A.
REVISIONS									OSCILLATOR AND OUTPUT CIRCUIT
UNLESS OTHERWISE SPECIFIED ALL DIMENSIONS IN INCHES				NEXT ASSEMBLY			BILL OF MATERIAL		
TOLERANCES									
BASIC DIAL.	FRACTIONS	DECIMALS							
UNDER 8	$\pm \frac{1}{64}$	$\pm .008$							
8 TO 24 INCL.	$\pm \frac{1}{32}$	$\pm .016$							
OVER 24	$\pm \frac{1}{16}$	$\pm .032$							
ANGLES $\pm$	ECCENTRICITY	TIR.							
HOLE DIA. $\pm$	SURFACES	✓							
COMMERCIAL TOLERANCES APPLY TO STOCK SIZES				SCALE	6	8 OF M	DATE	8-13-52	
				DRAWN	DES.	ENG.	APPRO.	B	
				FAS				2070287	
								A	
								REVISED JUL 1952	
								C	
								1	
								4	
								3	
								2	
								1	



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## FIGURE 14B

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	TOLERANCES		NEXT ASSEMBLY		BILL OF MATERIAL			
BASIC DIM.	FRACTIONS	DECIMALS						
UNDER 6	$\pm \frac{1}{64}$	$\pm .008$						
6 TO 24 INCL.	$\pm \frac{1}{32}$	$\pm .010$						
OVER 24	$\pm \frac{1}{16}$	$\pm .015$						
ANGLES $\pm$	OCENTRICITY TIR.		SCALE	DES.	S OF M	DATE		
HOLE DIA. $\pm$	SURFACES ✓							
COMMERCIAL TOLERANCES APPLY TO STOCK GEES			DRAWN	FAC	ENG.	APPROVED		
						B	2070298	A
						1		

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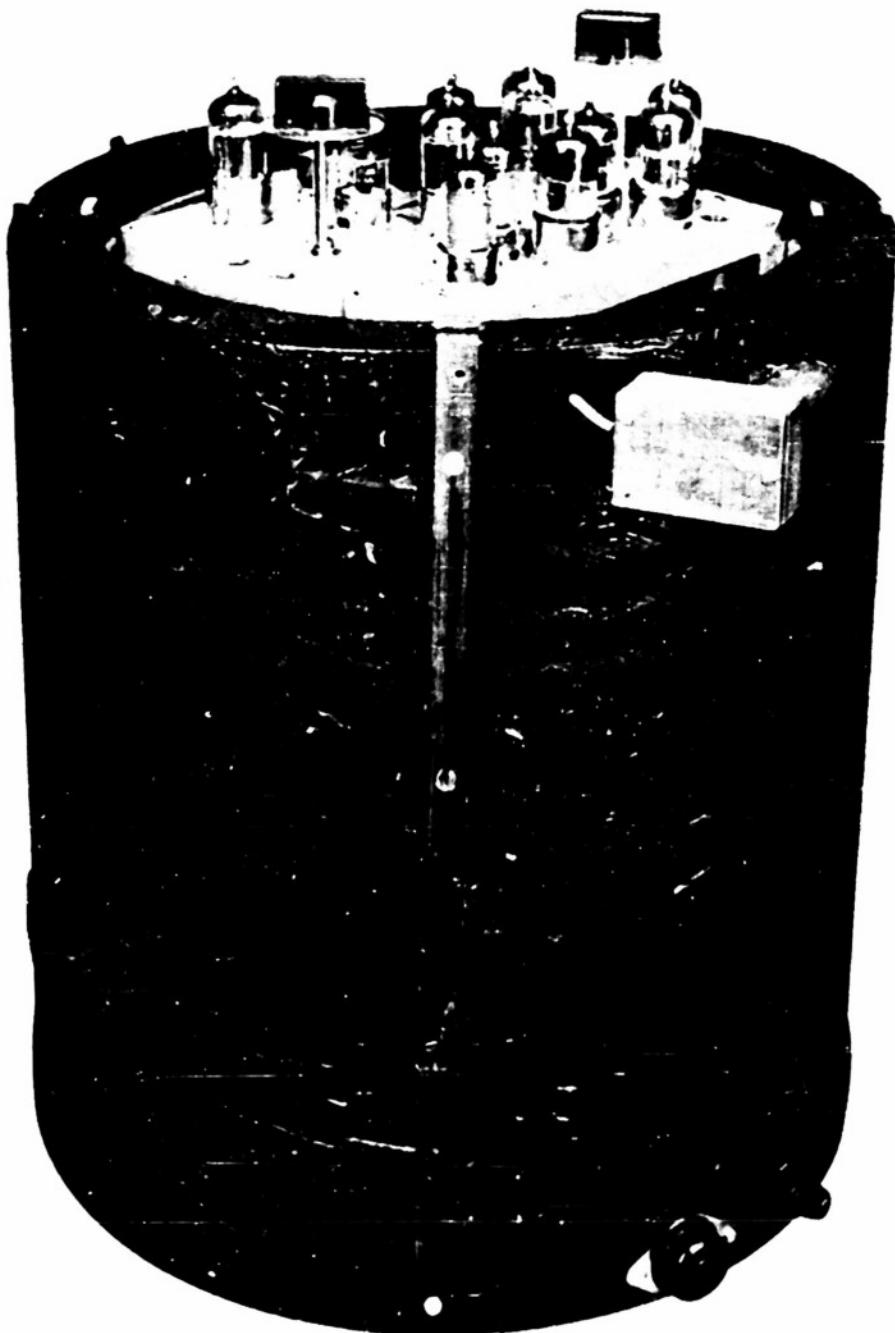


DELAY LINE AND POWER  
SUPPLY - TOP VIEW

FIG. 15  
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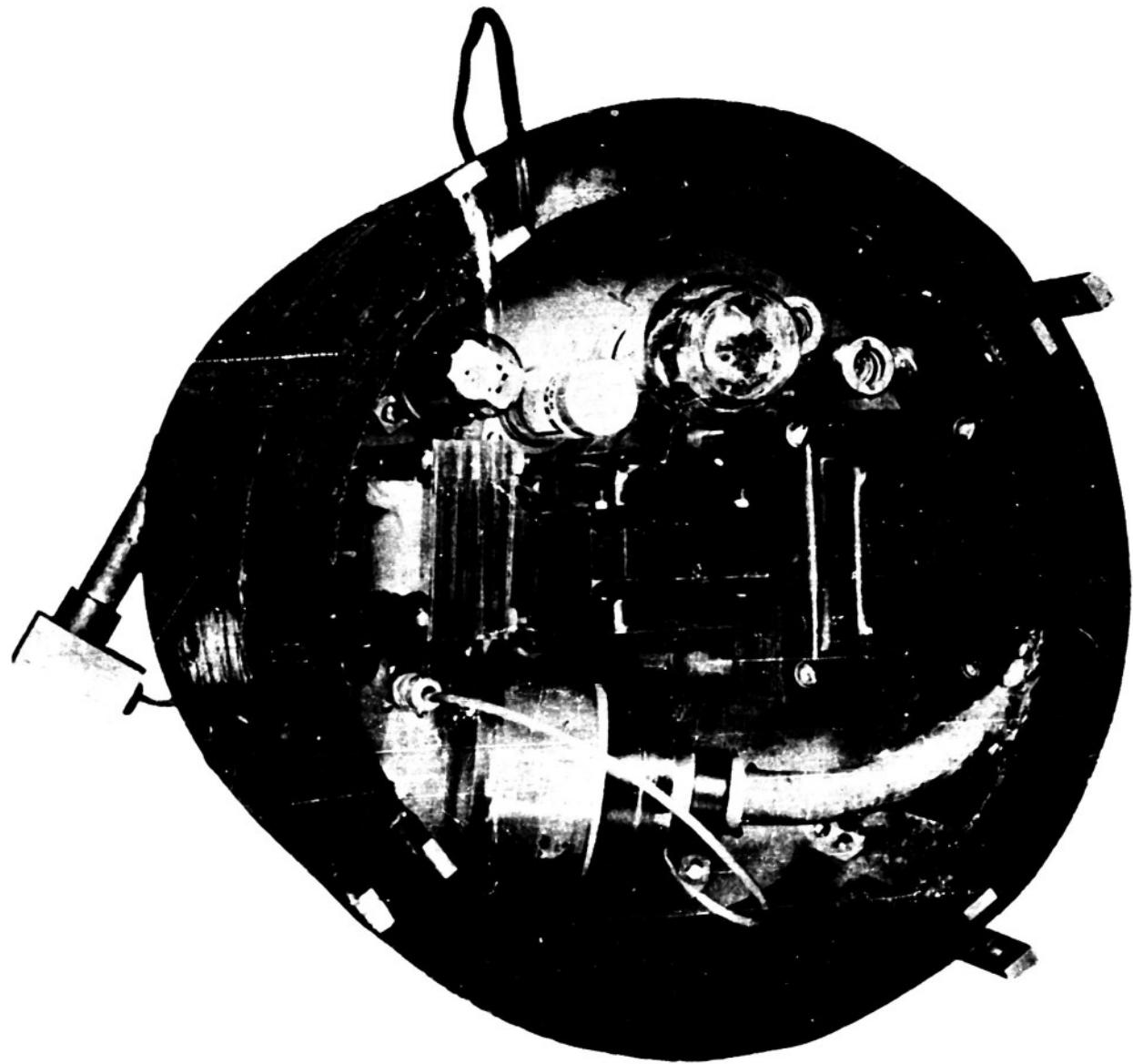
**COMB FILTER - SIDE VIEW**

**FIG. 16**

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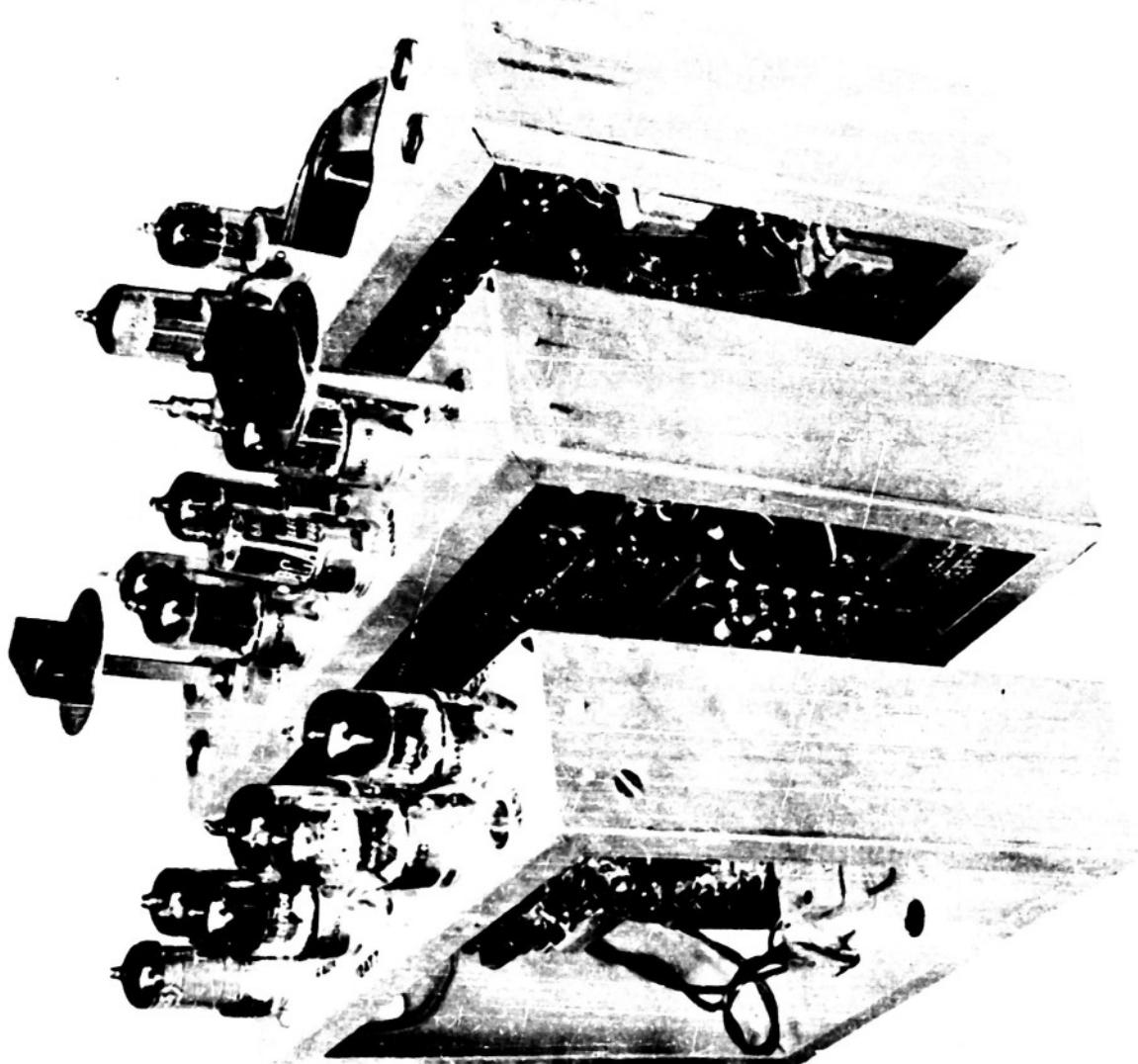
COMB FILTER-BOTTOM VIEW

FIG. 17

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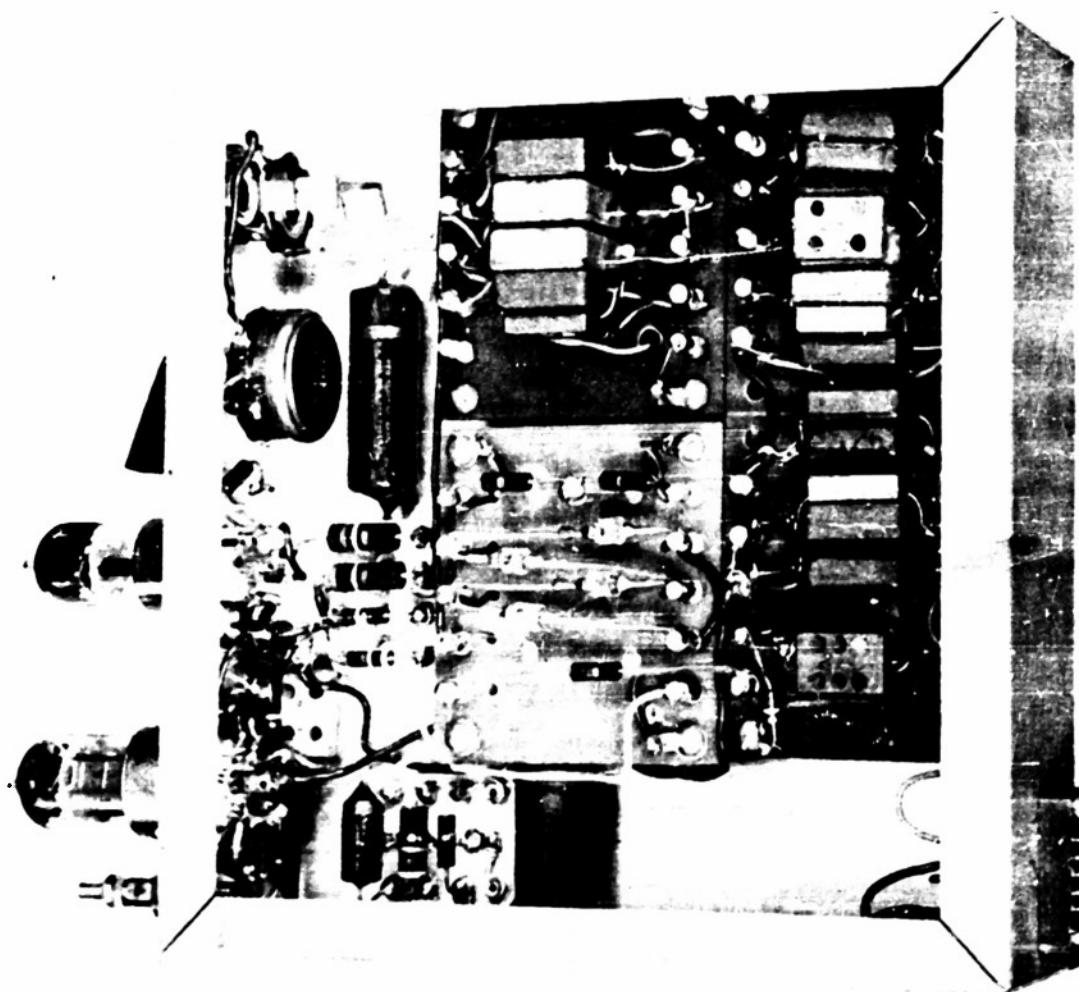


**PLUG-IN UNITS**  
**FIG. 18**

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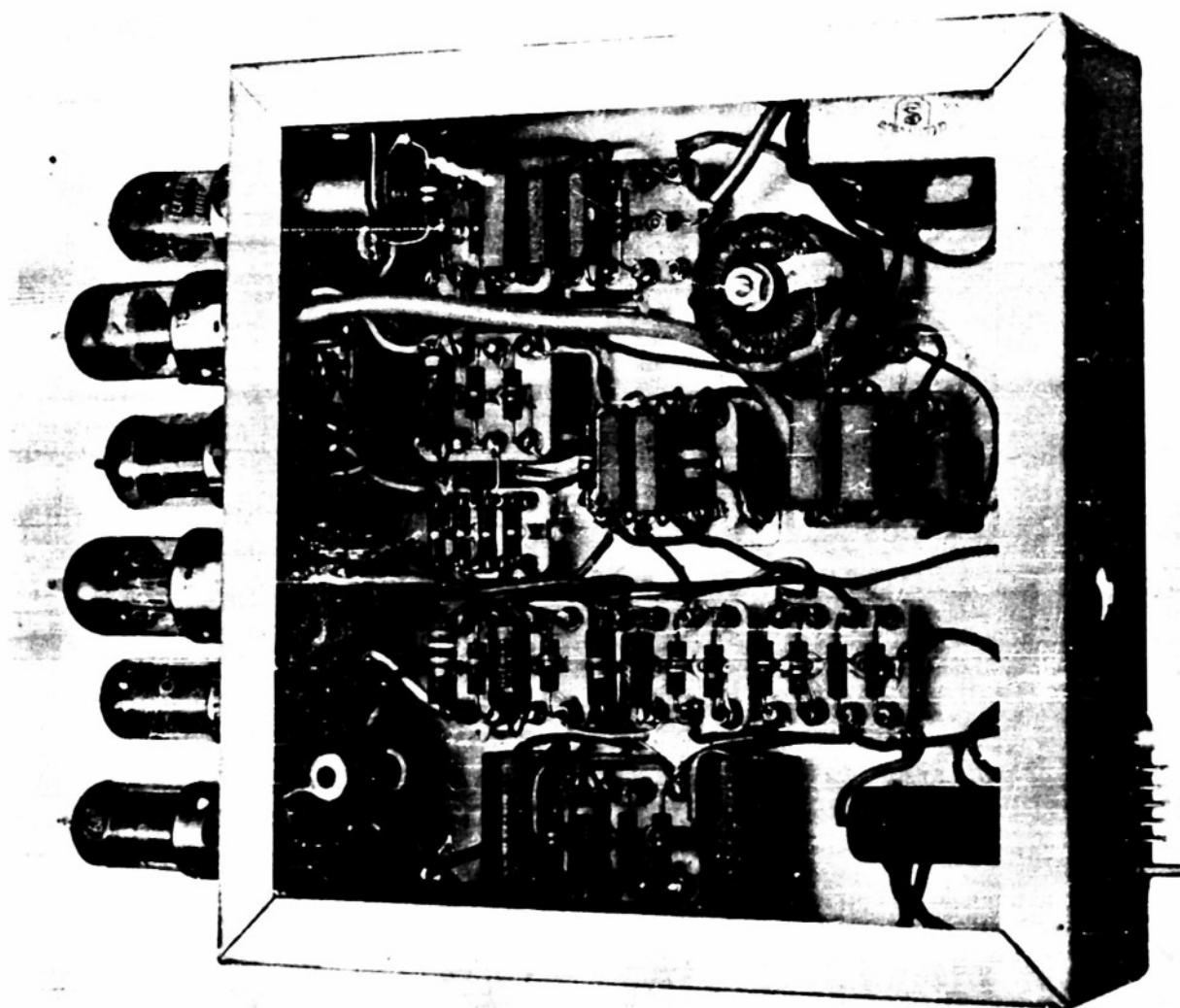


INPUT CHASSIS  
FIG. 19

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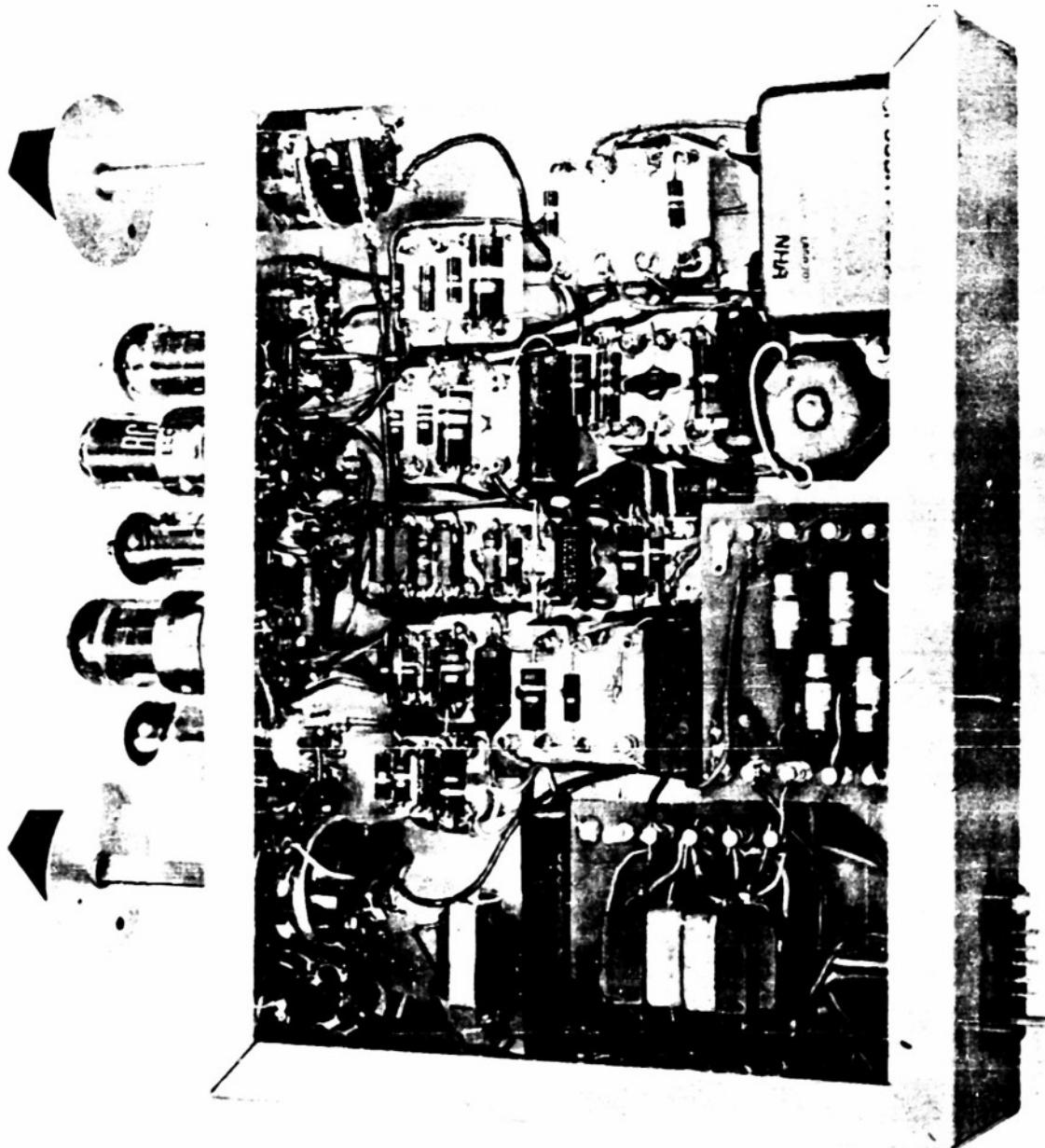
**FEEDBACK AMPLIFIER**

**FIG. 20**

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**OSCILLATOR AND OUTPUT  
CHASSIS**

**FIG. 21**  
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COMB FILTER LISTENING TEST

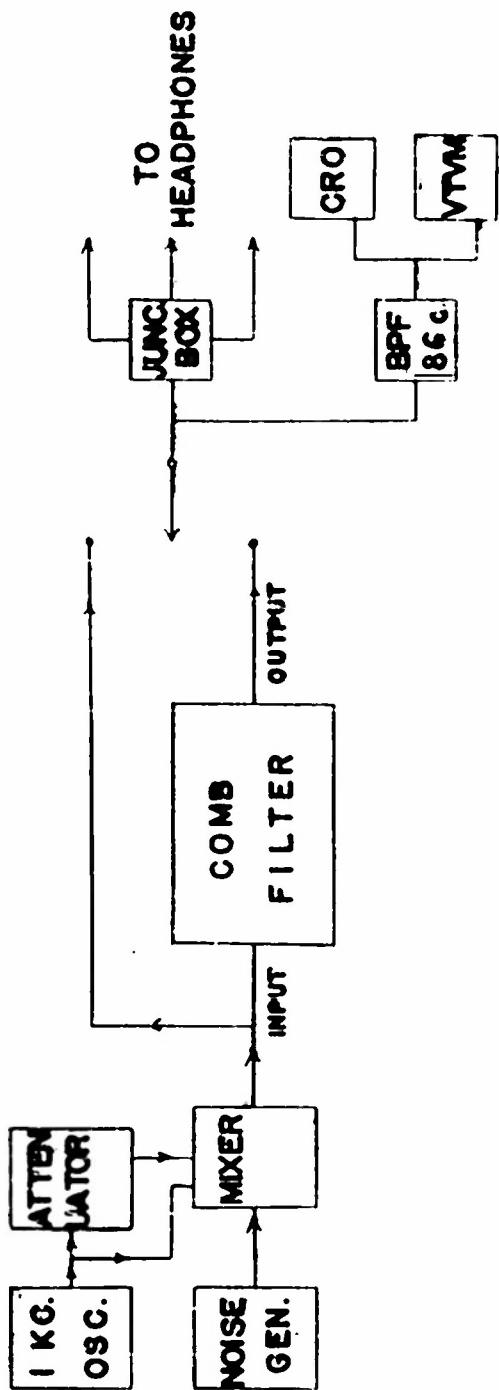


FIGURE 22

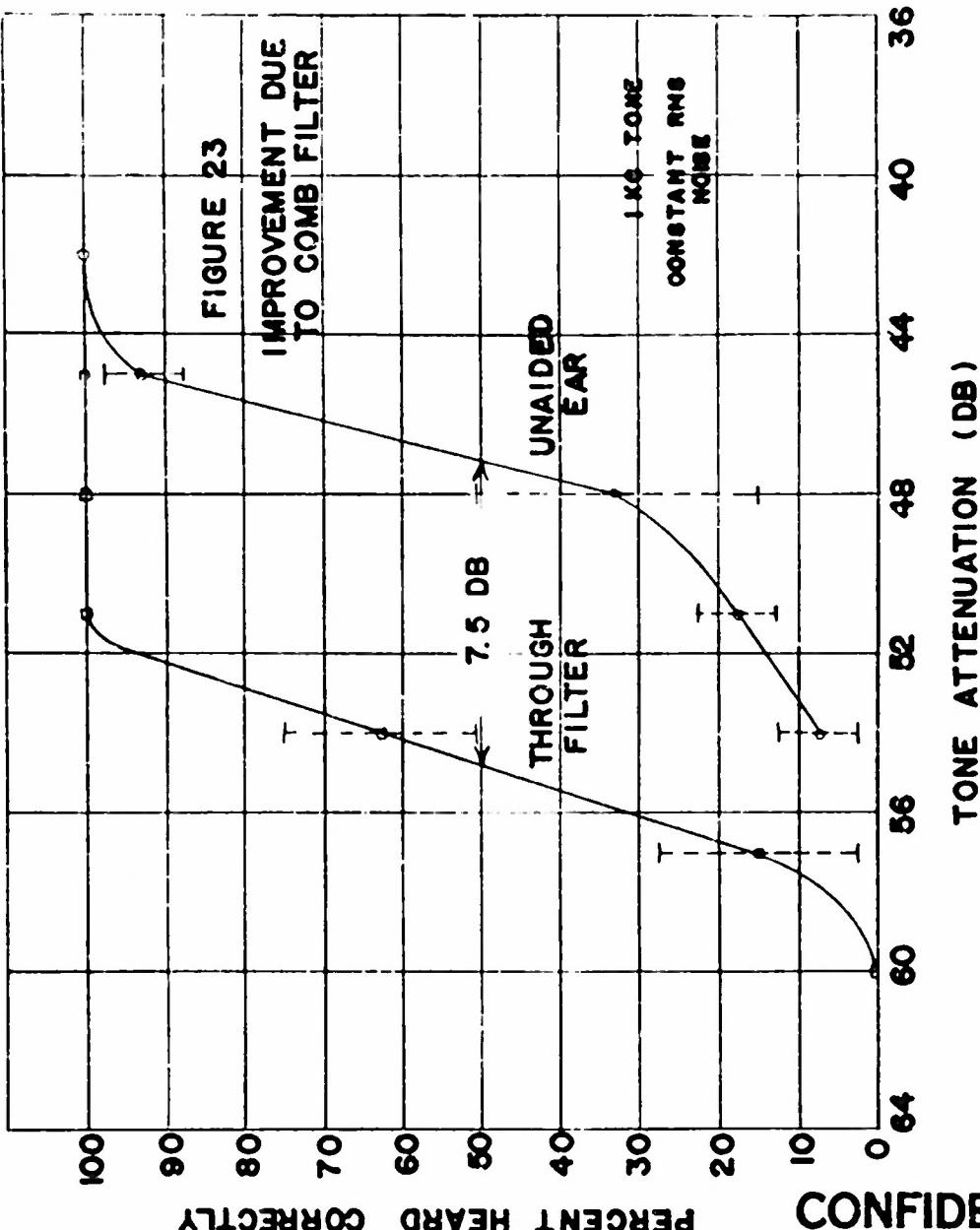
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TOLERANCE UP TO ABOVE ABOVE	MATERIAL	TITLE			
DEC. DIM. $\pm .008$ $\pm .010$ $\pm .015$	FINISH	ISSUED	USED WITH	APPD	DWN.
FRACT. DIM. $\pm \frac{1}{64}$ $\pm \frac{1}{32}$ $\pm \frac{1}{16}$ UNLESS OTHERWISE SPECIFIED					

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FED. REP. NO. 100-1001-100

TOLERANCE	MATERIAL	TITLE			
UP TO .005 ABOVE .010					
DEC. DIM. $\pm .005$	$\pm .010$	$\pm .015$			
FRACT. DIM. $\pm \frac{1}{64}$	$\pm \frac{1}{32}$	$\pm \frac{1}{16}$			
UNLESS OTHERWISE SPECIFIED			ISSUED	USED WITH	APPD DWN.
Federal Telecommunication Laboratories, Inc.			-1		

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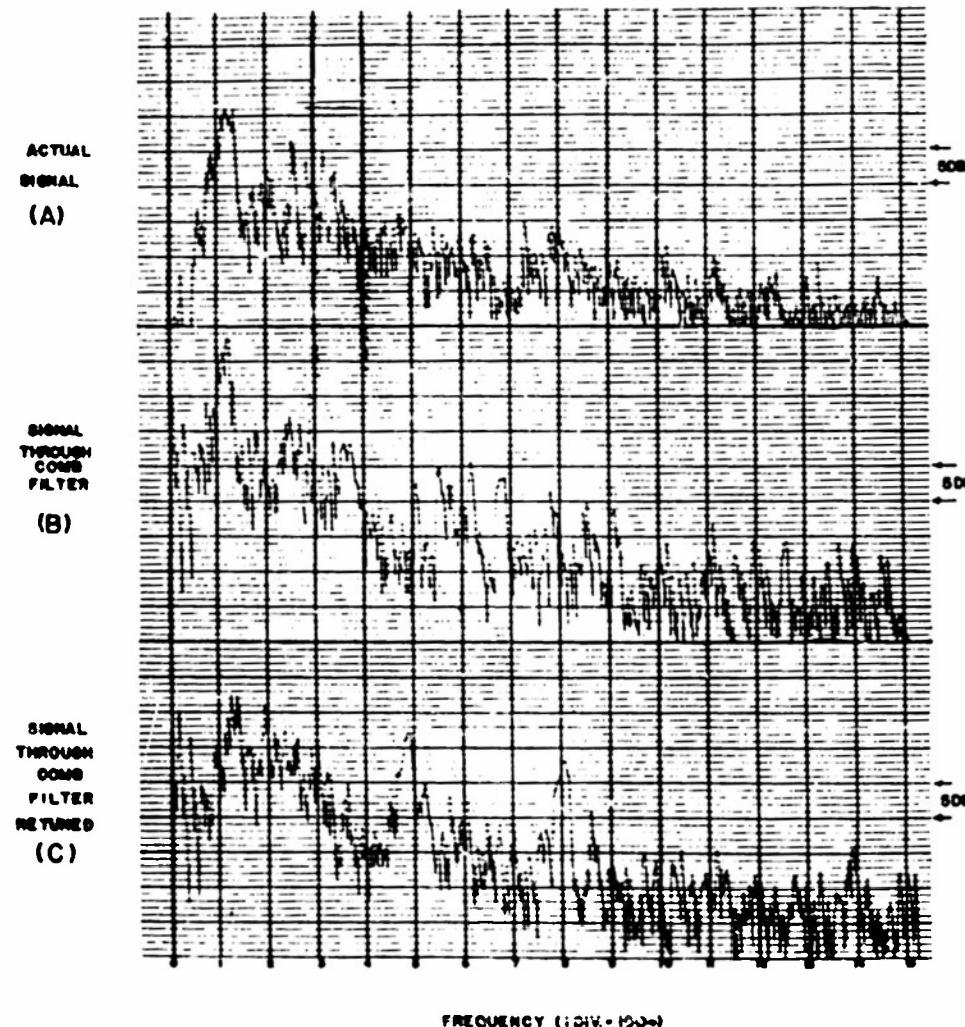
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**USE OF COMB FILTER IN DETECTION OF TONES**

TARGET IS U.S. TUG AT 10,000 YDS. MAKING 200 TURNS  
LISTENING VESSEL IS U.S. FLYING FISH

ANALYZER BANDWIDTH OF 20Hz



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**FIGURE 24**

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USE OF THE COMB FILTER IN DETECTION OF TONES

SELF NOISE OF U.S.S. HALIFAX  
WHILE HOVERING (DAY HYDROPHONE)

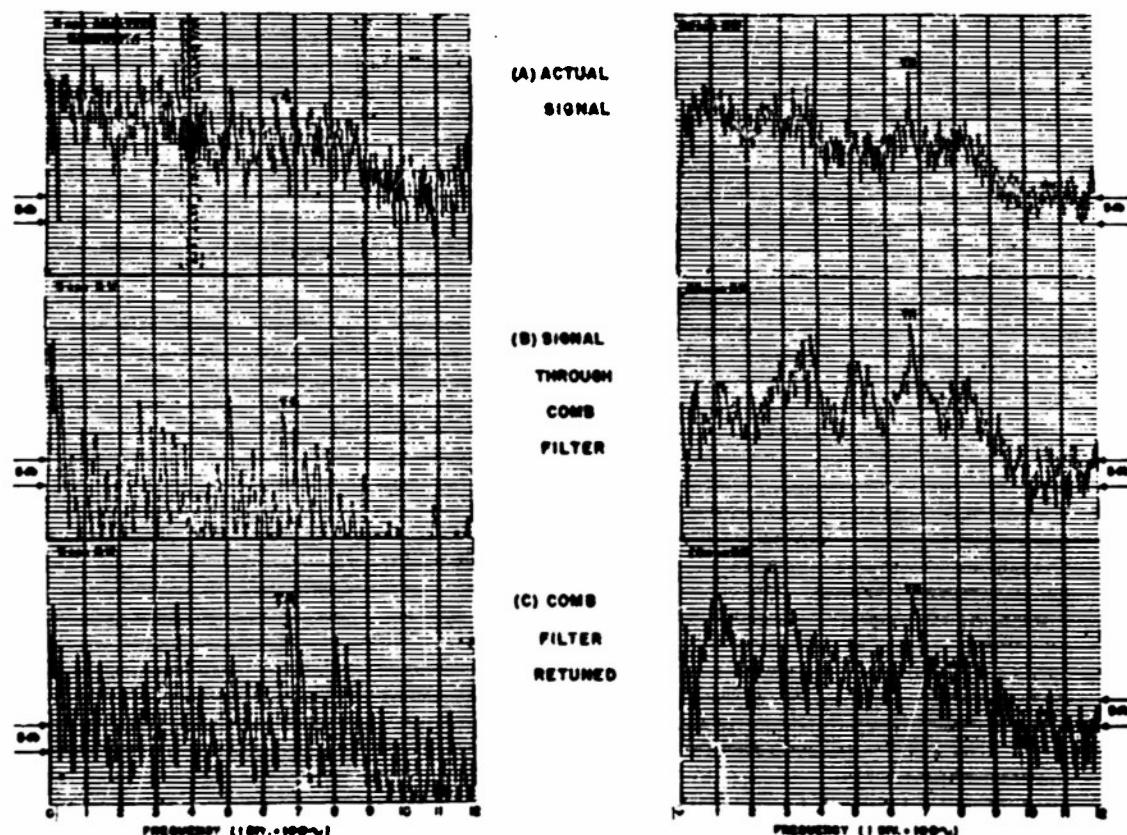
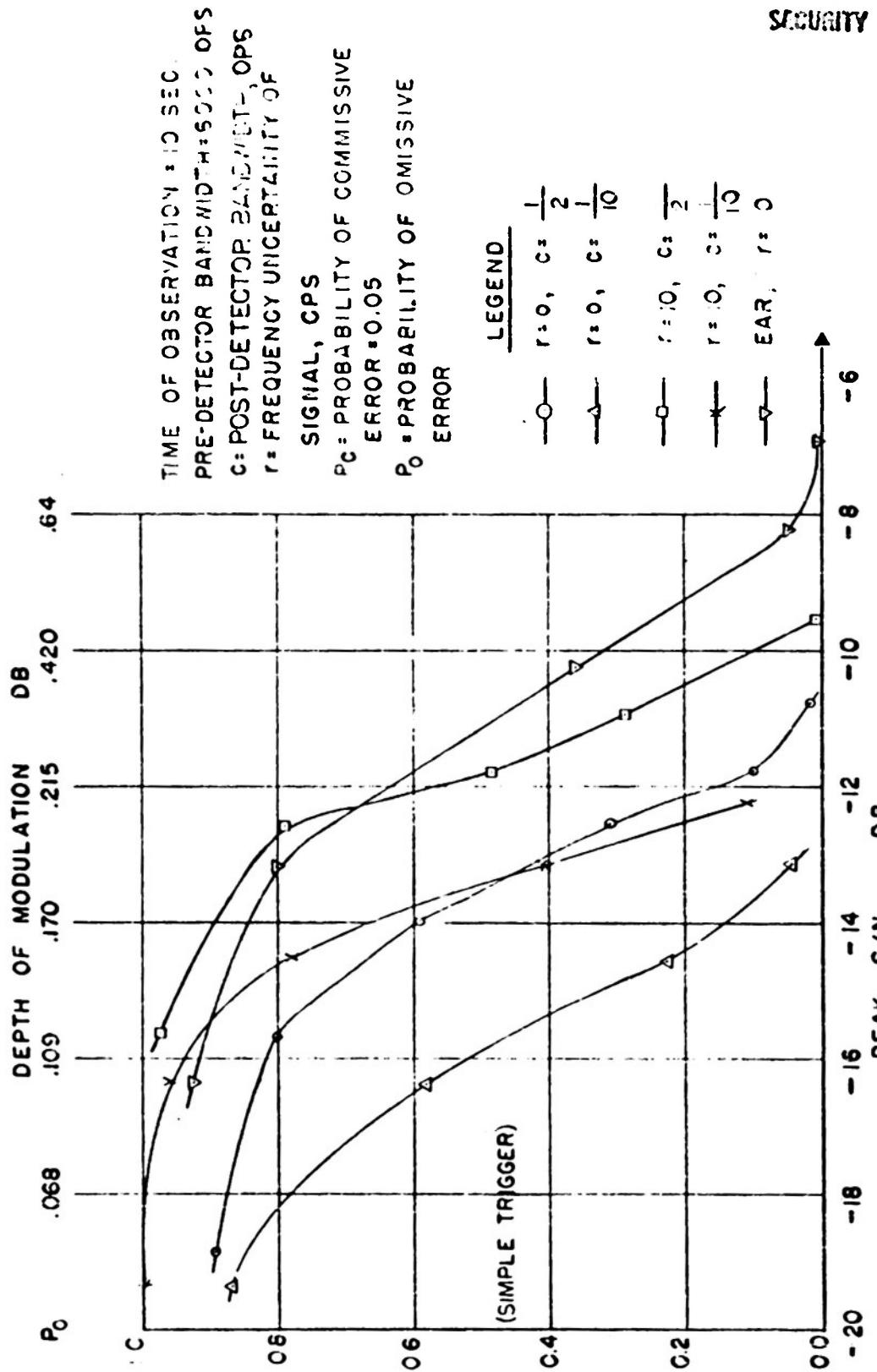


FIGURE 25

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DETECTION OF AN AMPLITUDE MODULATED WHITE NOISE IN A WHITE NOISE BACKGROUND.  
FIGURE 26

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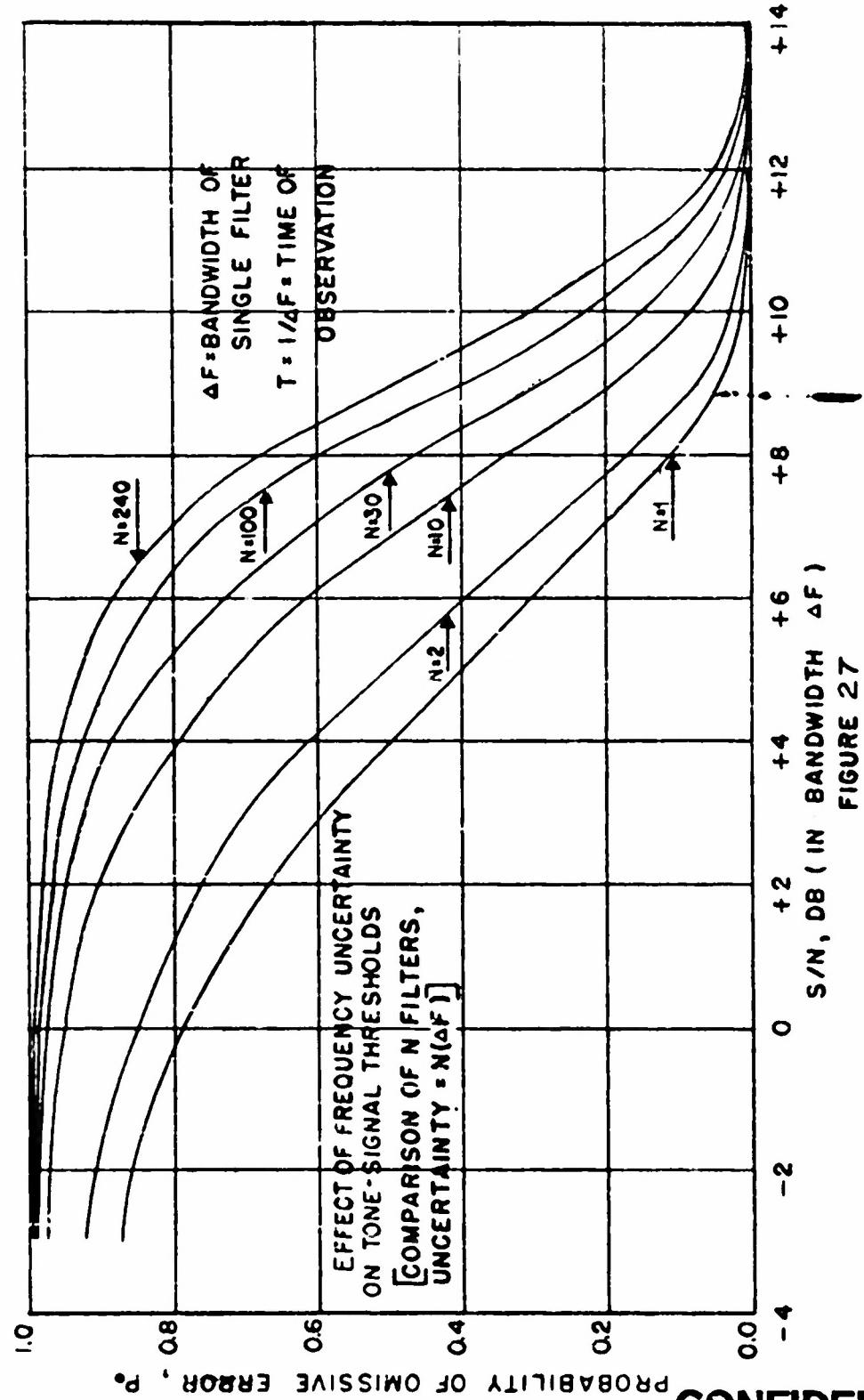


FIGURE 27

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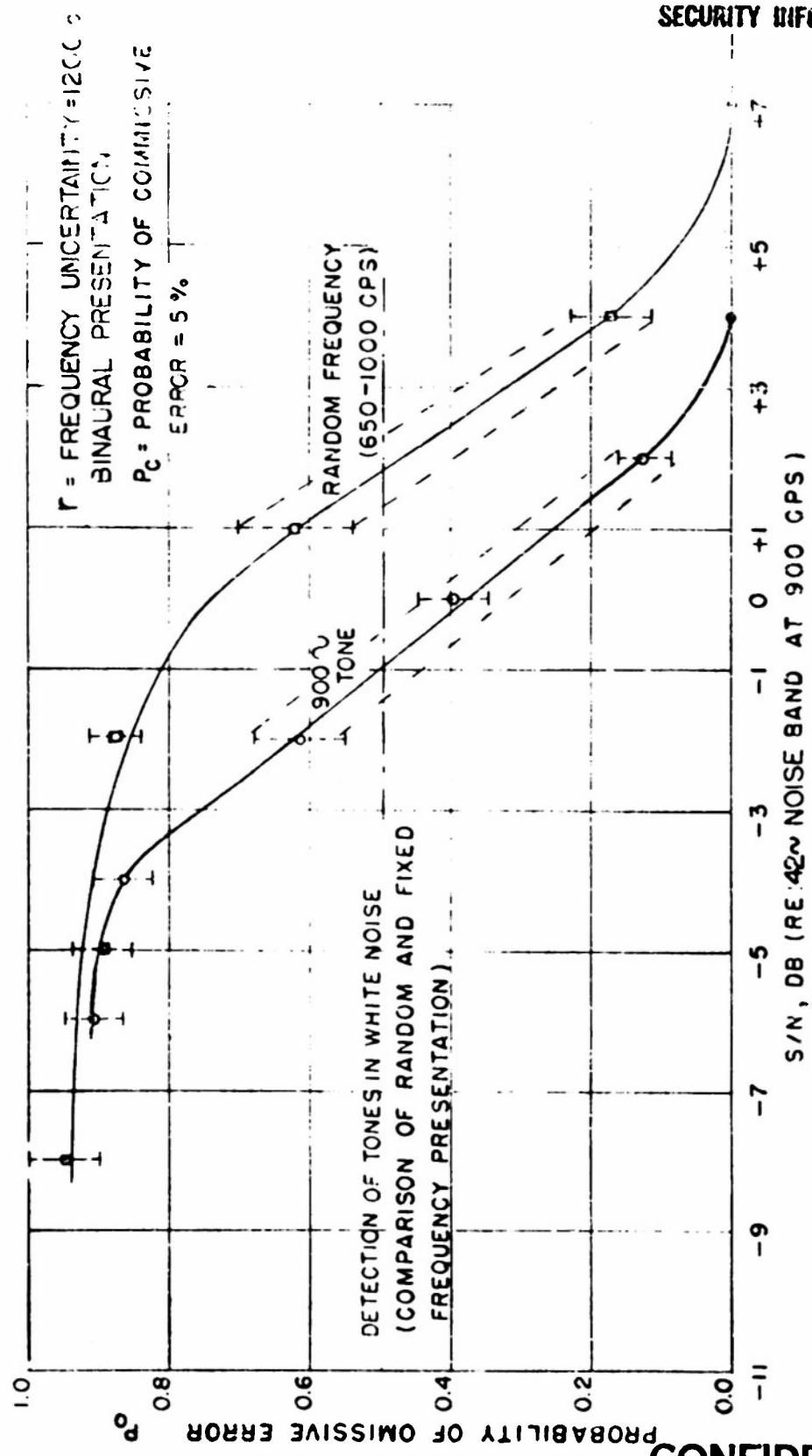
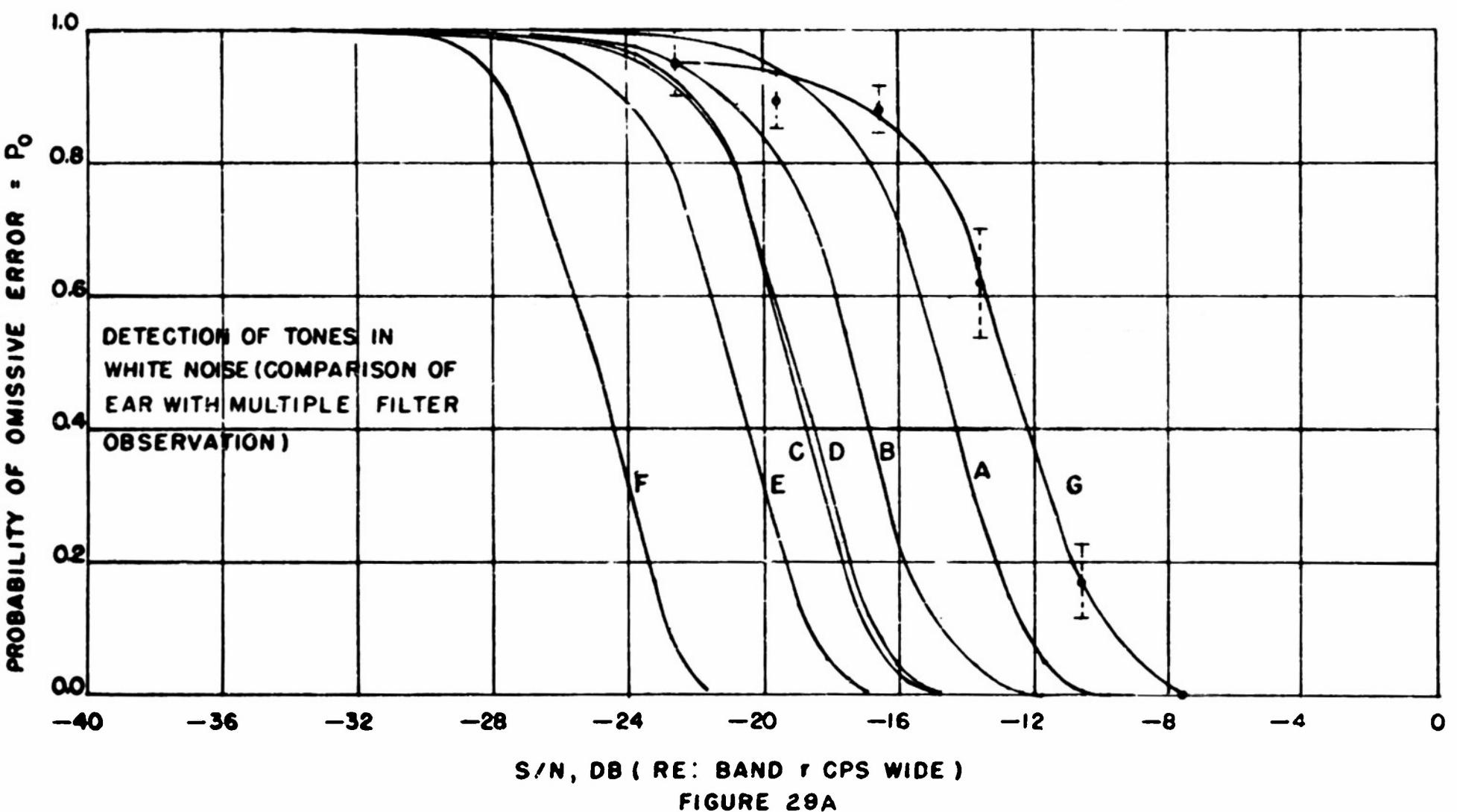


FIGURE 28

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$T_0$  = REFERENCE TIME OBSERVATION, SEC.  
 $T$  = TIME OF OBSERVATION, SEC.  
 $C$  = BANDWIDTH OF SINGLE FILTER, CPS  
 $C_0$  =  $1/T_0$ .  
 $r$  = FREQUENCY UNCERTAINTY OF SIGNAL, CPS  
 $H$  = NO. OF OBSERVATIONS OF A SINGLE FILTER

LEGEND

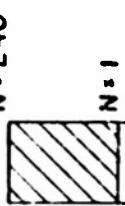
A)	$T = T_0$ ,	$C = C_0$ ,	$H = 1$ ,	$r = 240C$
B)	$T = 3T_0$ ,	$C = C_0$ ,	$H = 3$ ,	$r = 240C$
C)	$T = 5T_0$ ,	$C = C_0$ ,	$H = 5$ ,	$r = 240C$
D)	$T = 3T_0$ ,	$C = C_0/3$ ,	$H = 1$ ,	$r = 720C$
E)	$T = 5T_0$ ,	$C = C_0/5$ ,	$H = 1$ ,	$r = 1200C$
F)	$T = 25T_0$ ,	$C = C_0/5$ ,	$H = 5$ ,	$r = 1200C$
G)	EAR,	$T = 5$ SEC,	$r = 1200 \sim$	

(WHEN  $H > 1$ , MAJORITY REPORT IS USED)

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NUMBER OF OBSERVATIONS,  $N$



DETECTION OF TONES IN WHITE  
NOISE. IMPROVEMENT DUE TO  
TAKING MAJORITY REPORT OF  $N$   
OBSERVATIONS.  
(RE: S/N FOR  $N=1$ , AT 50% RD)  
 $N$  = NO. OF FILTERS

FIGURE 29B

$N = 240$

$N = 1$

$N = 240$



FM 100-100-1000

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DETECTION OF A TONE IN A BAND OF NOISE

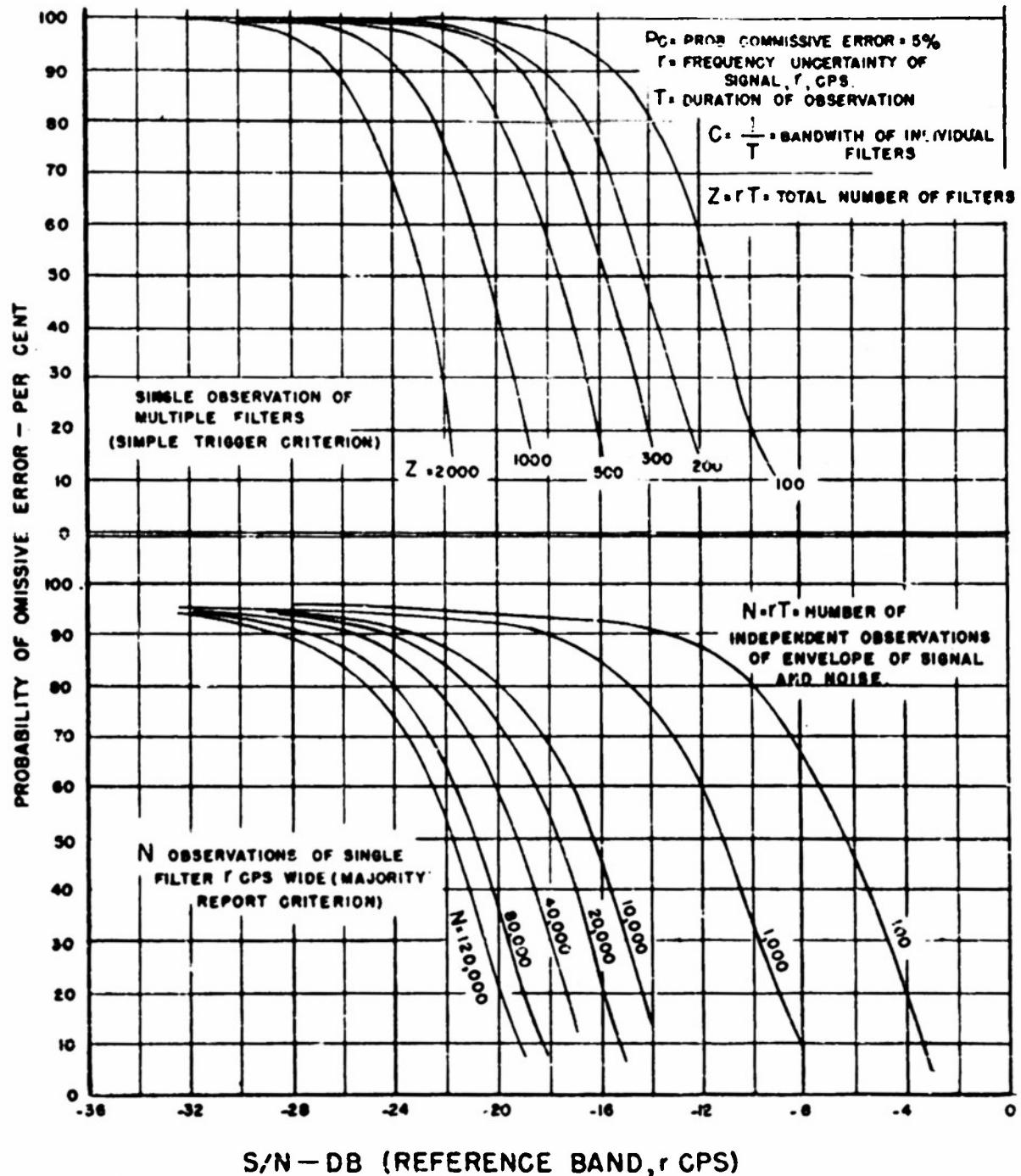
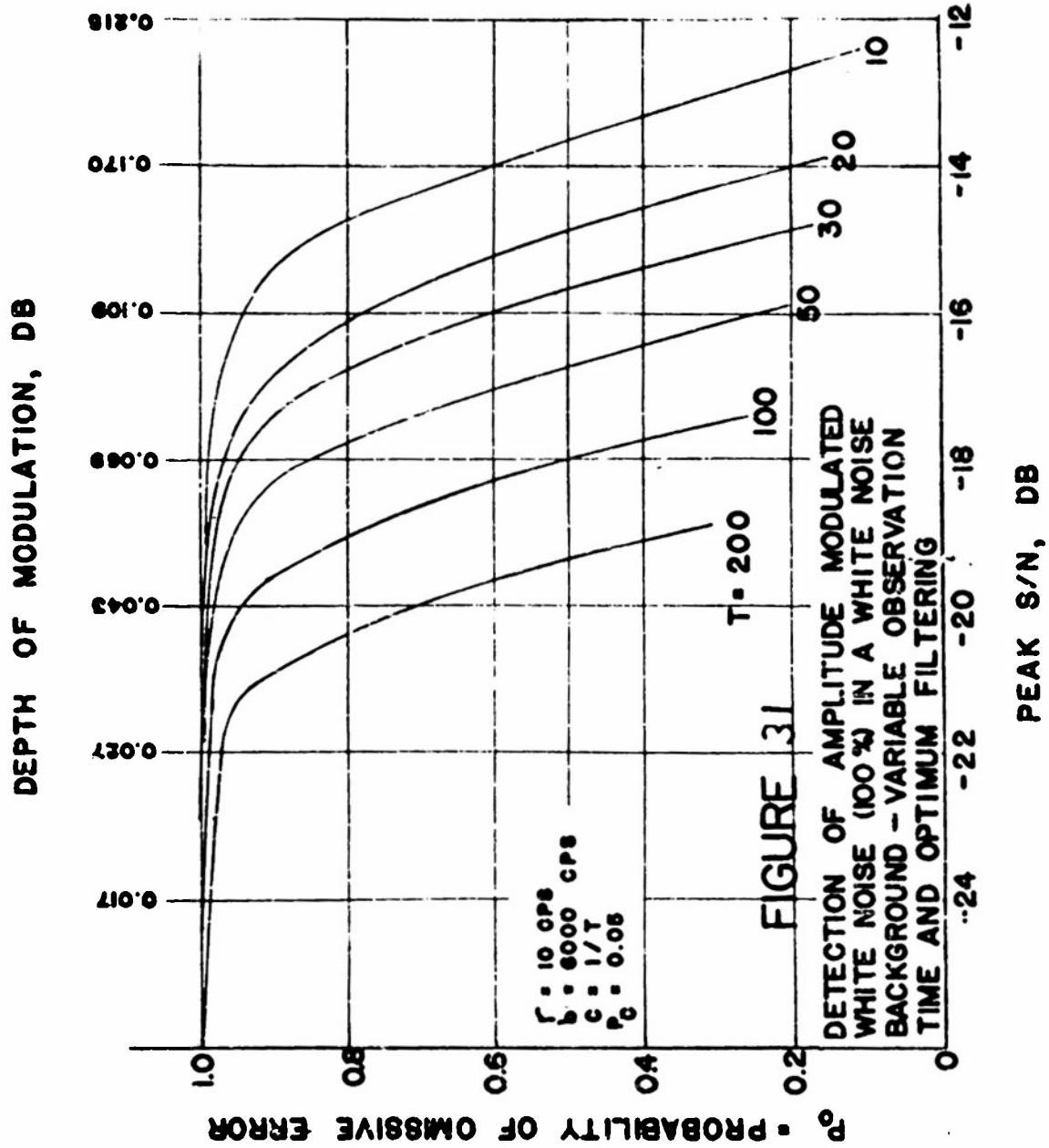


FIGURE 30

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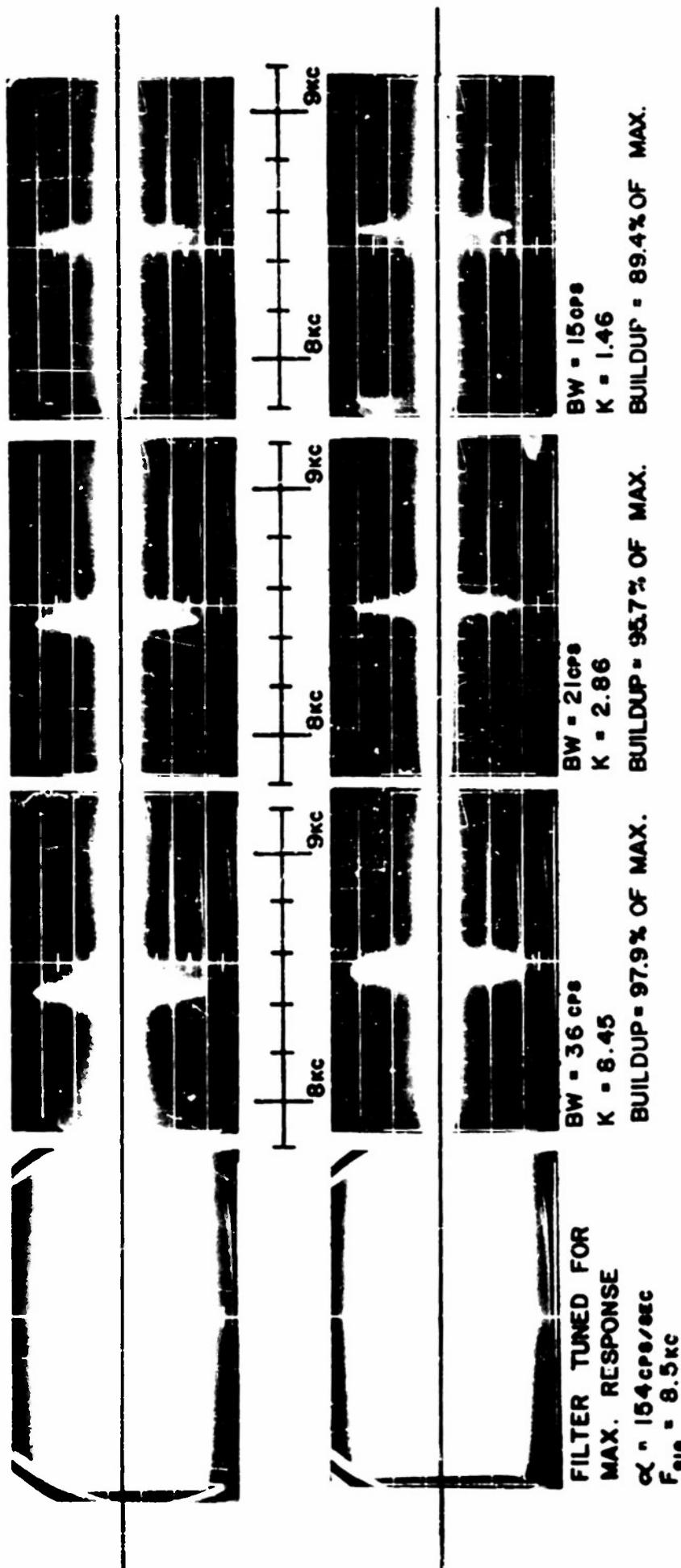
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**SECURITY INFORMATION**



PRT. NO. 1000-142(00)

TOLERANCE	MATERIAL	TITLE
UP TO ABOVE ABOVE		
DEC. DIM. ± .000 ± .000 ± .010 ± .010		
FRACT. DIM. ± $\frac{1}{32}$ ± $\frac{1}{32}$ ± $\frac{1}{16}$ ± $\frac{1}{16}$	FINISH	ISSUED
UNLESS OTHERWISE SPECIFIED	<b>CONFIDENTIAL</b> <b>SECURITY INFORMATION</b>	
<i>Federal Telecommunication Laboratories, Inc.</i>		-1

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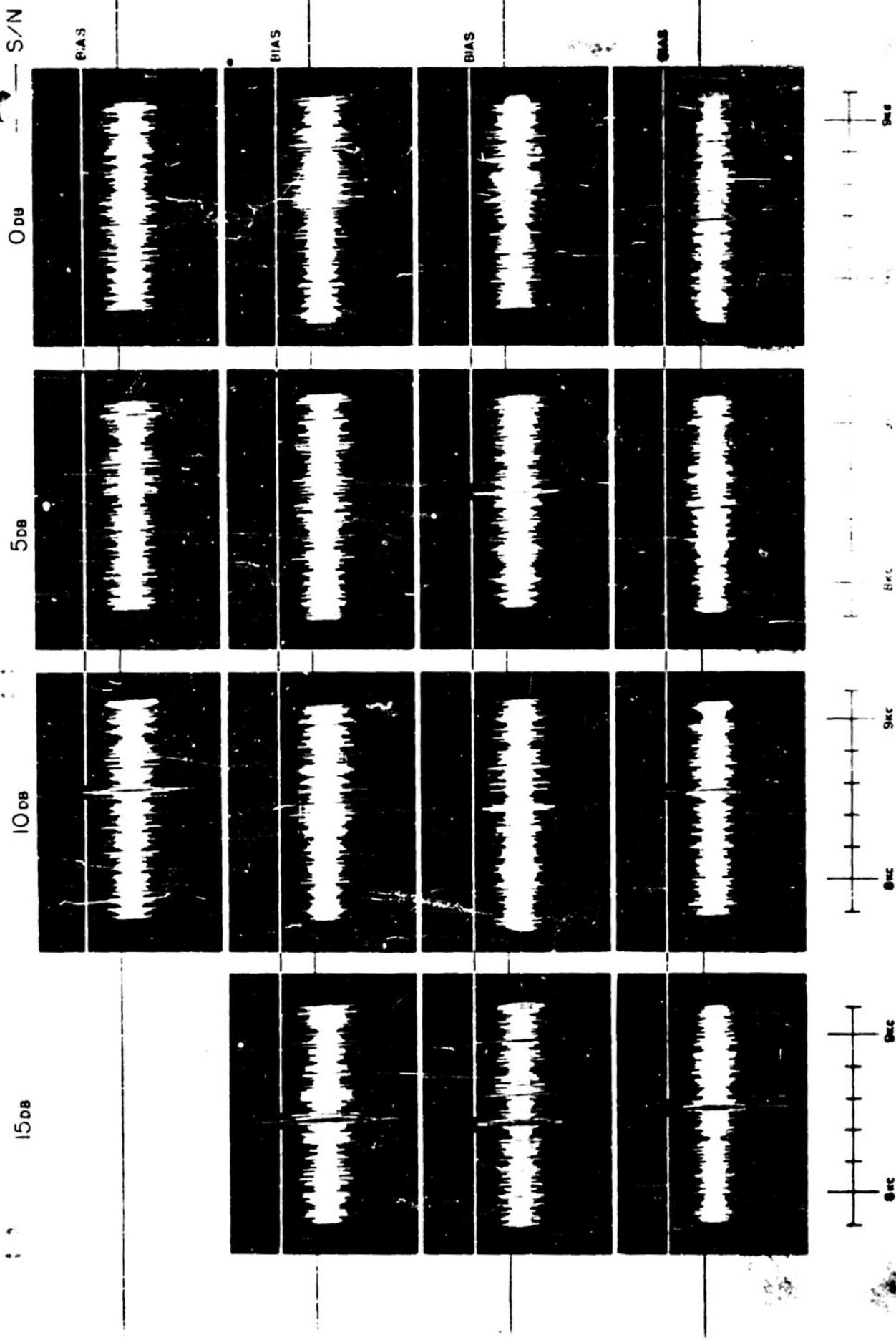


BUILDUP OF A TONE  
IN A FREQUENCY  
SCANNING ANALYZER

FIGURE 32

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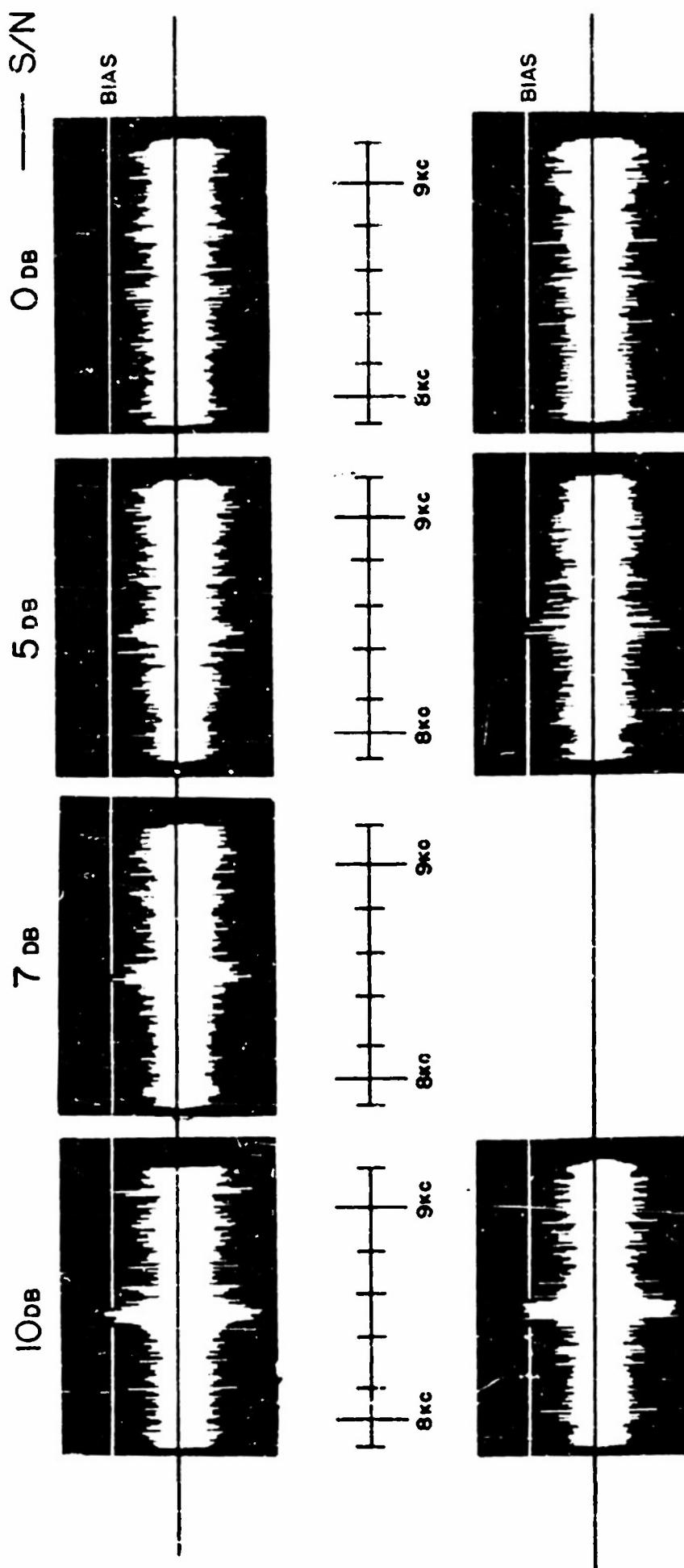
DETECTION OF A TONE IN  
NOISE BY A FREQUENCY  
SCANNING ANALYZER

FREQUENCY = 15 cps  
TIME = 1.46 sec  
AMPLITUDE = -0.3 mV  
MEASURED IN 10db BAND

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FIGURE 33

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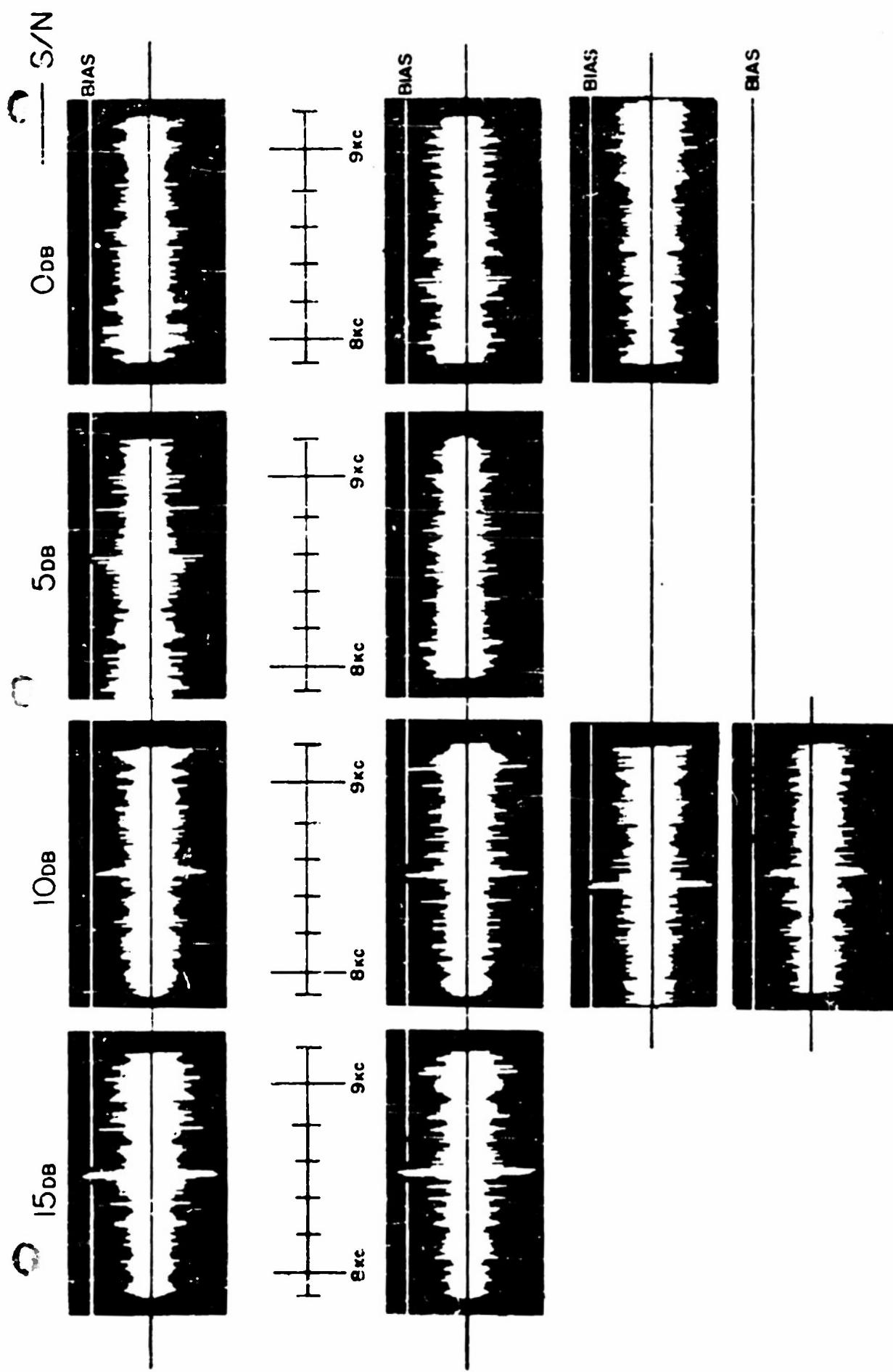
DETECTION OF A TONE IN  
NOISE BY A FREQUENCY  
SCANNING ANALYZER

BW = 36 cps  
 $\alpha$  = 154 cps/sec  
K = 8.45  
F<sub>cue</sub> = 8.5 kc  
S/N MEASURED IN 36 cps BAND

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FIGURE 34

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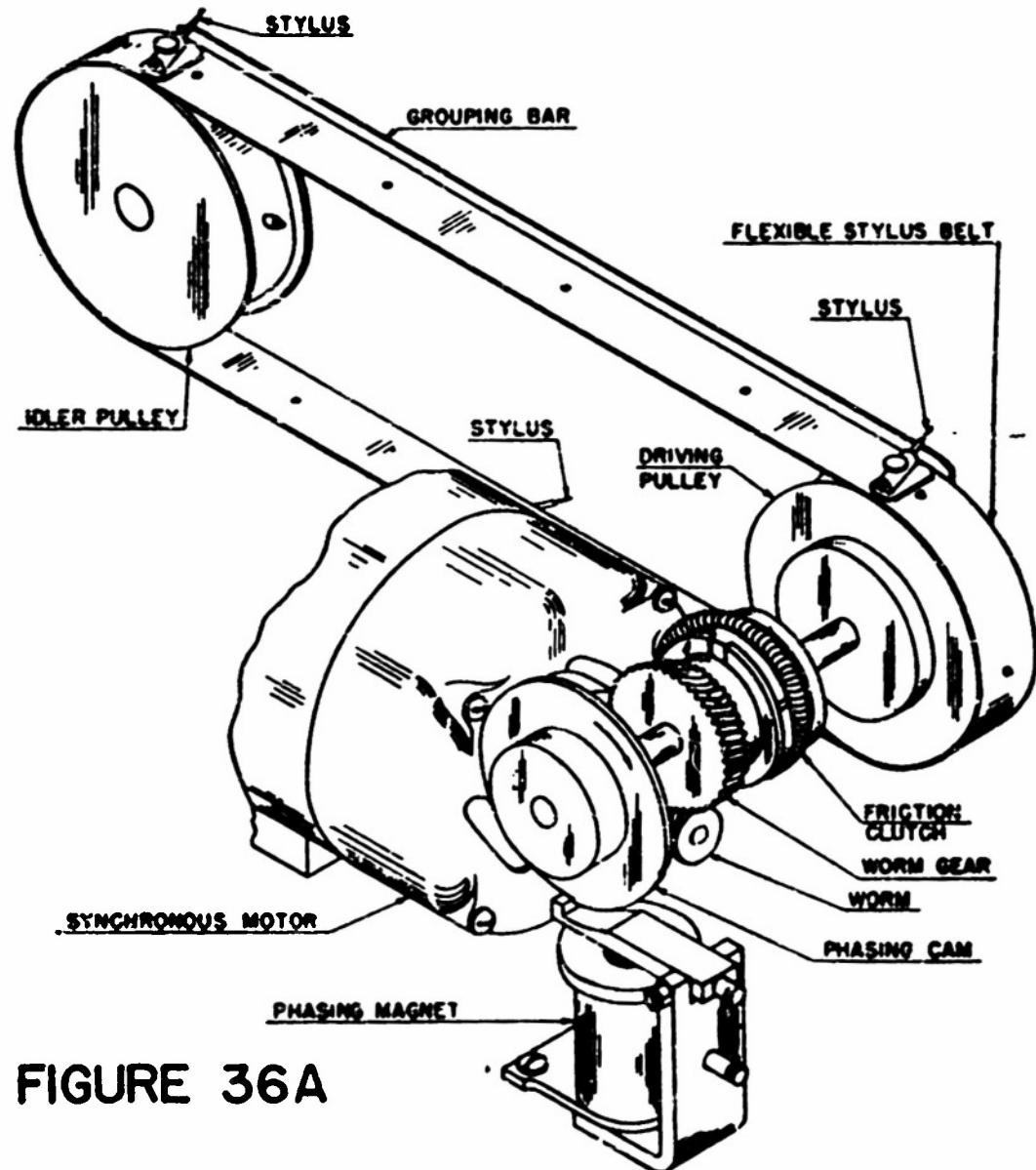
DETECTION OF A TONE IN  
NOISE BY A FREQUENCY  
SCANNING ANALYZER

BW = 21 cps  
K = 2.96  
dc = 154 cps/sec  
F<sub>sig</sub> = 8.5 kc  
S/N MEASURED IN 21 cps BAND

FIGURE 3.5

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**FIGURE 36A**

**FACSIMILE RECORDER  
STYLUS DRIVE MECHANISM**

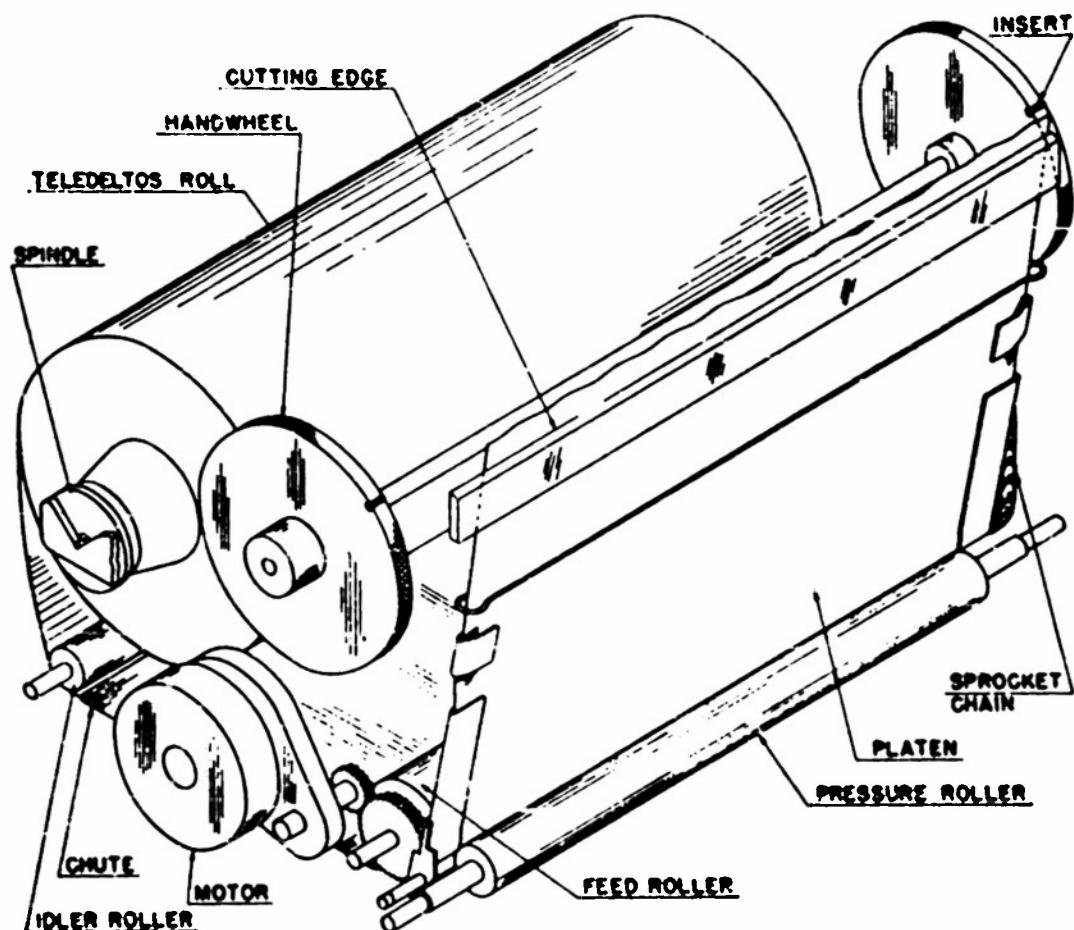
Courtesy of Western Union Review

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## FACSIMILE RECORDER



PAPER FEED MECHANISM  
FIGURE 36 B

Courtesy of Western Union Review

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~~-2 DB~~

~~1 KC SINE WAVE - 2 DB STEPS~~

~~40 CPS BAND OF NOISE AT 1 KC - 5 DB STEPS~~

~~-5 DB~~

**RESPONSE OF RECORDER**  
**FIGURE 37 A**

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**FIGURE 37 B**

**DETECTION OF PULSED TONES IN NOISE**

8/N<sup>a</sup>  
8 DB - 10

8 DB

8

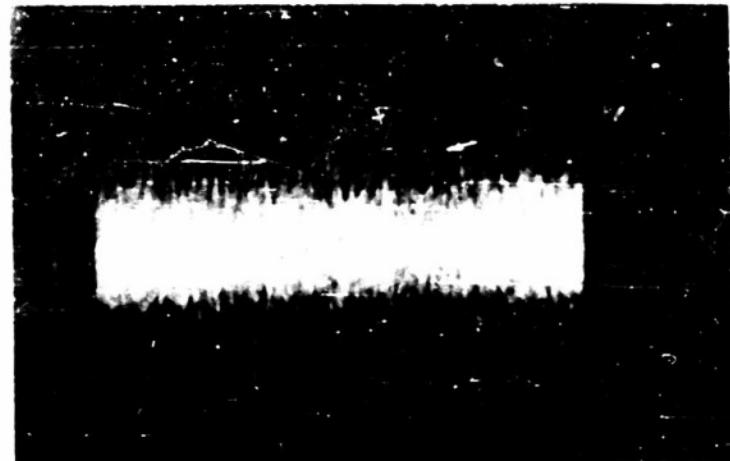
10

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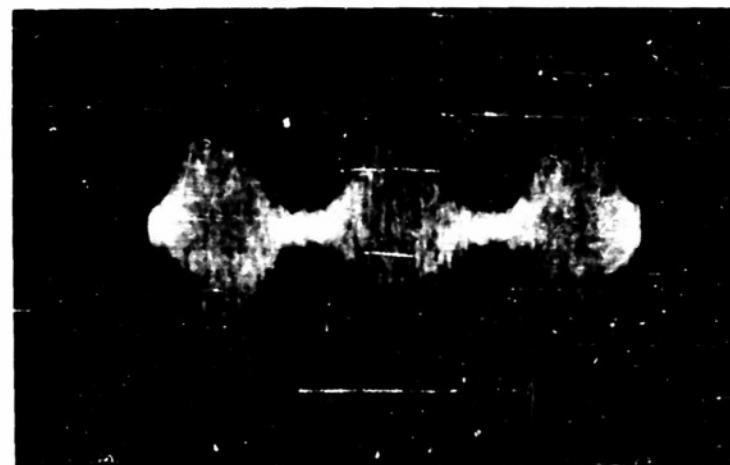
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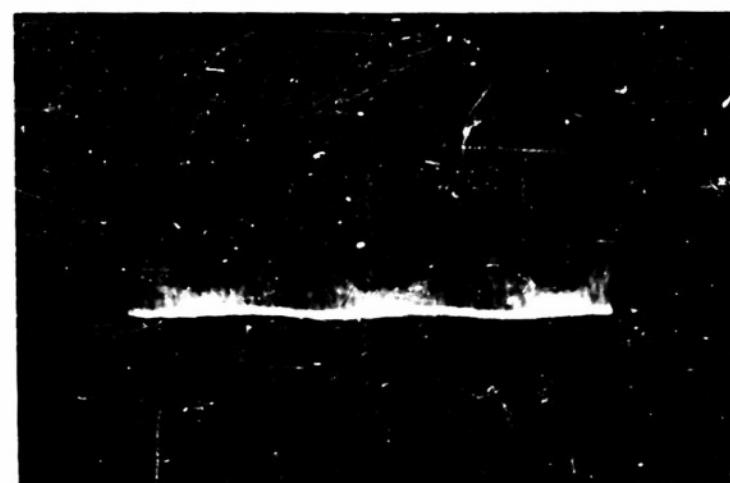
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NOISE BACKGROUND



AMPLITUDE MODULATED TARGET  
NOISE



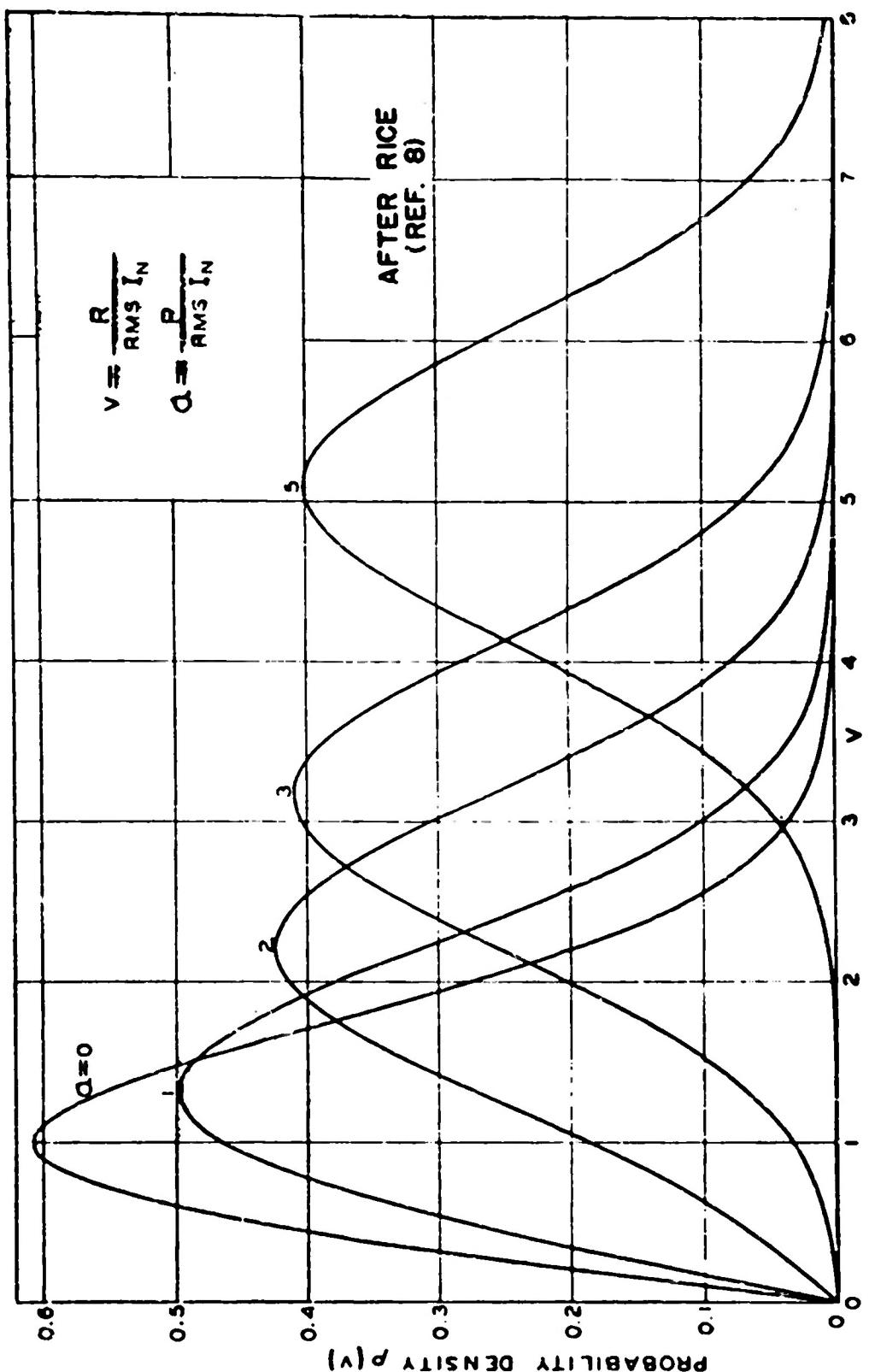
TARGET INTENSITY

**FIGURE 38**

OSCILLOGRAMS OF  
IDEALIZED BACK -  
GROUND AND TARGET  
NOISES

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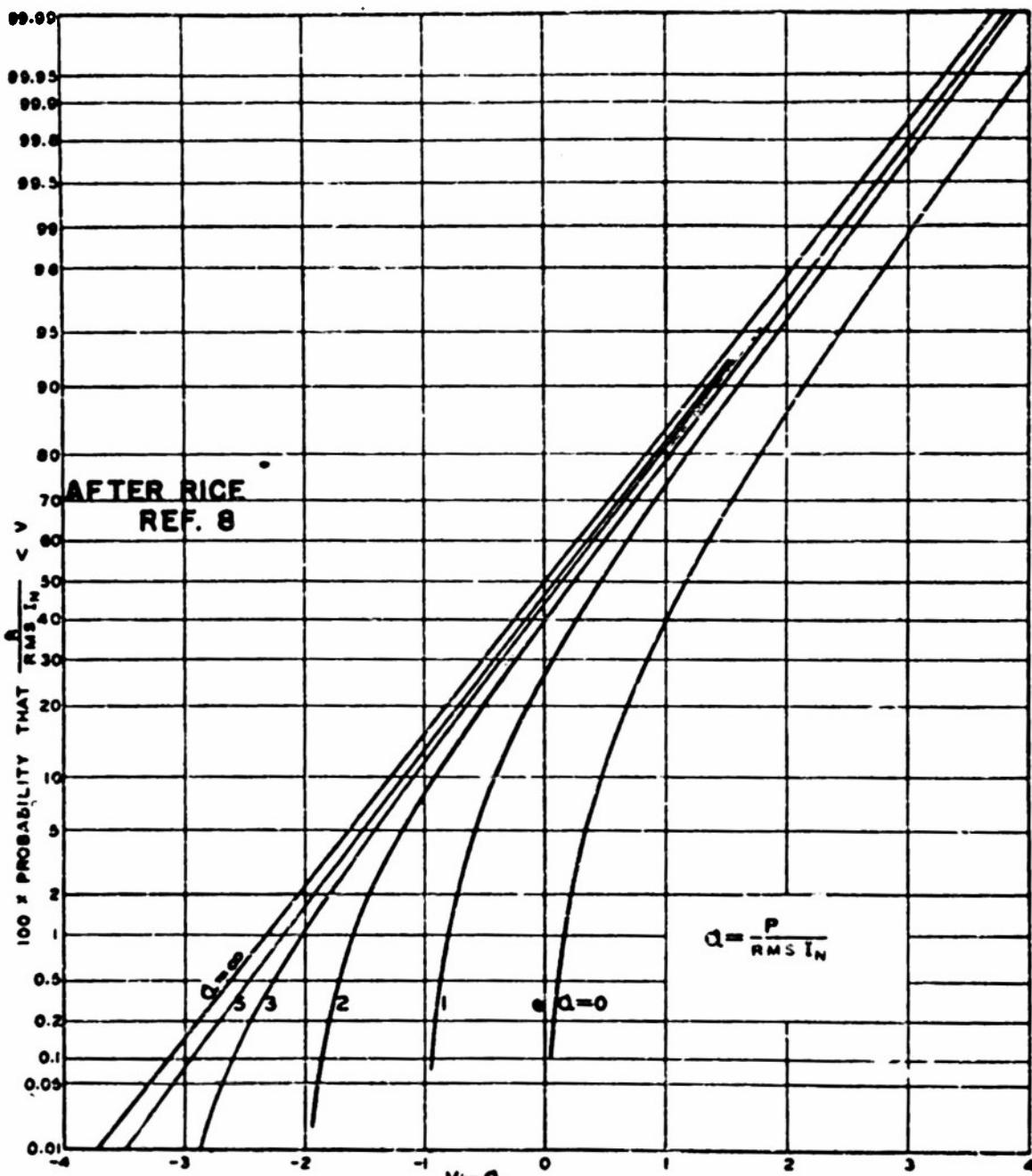


Probability density of envelope  $R$  of  $I(t) = P \cos pt + I_N$

FIGURE 39

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**FIGURE 40**

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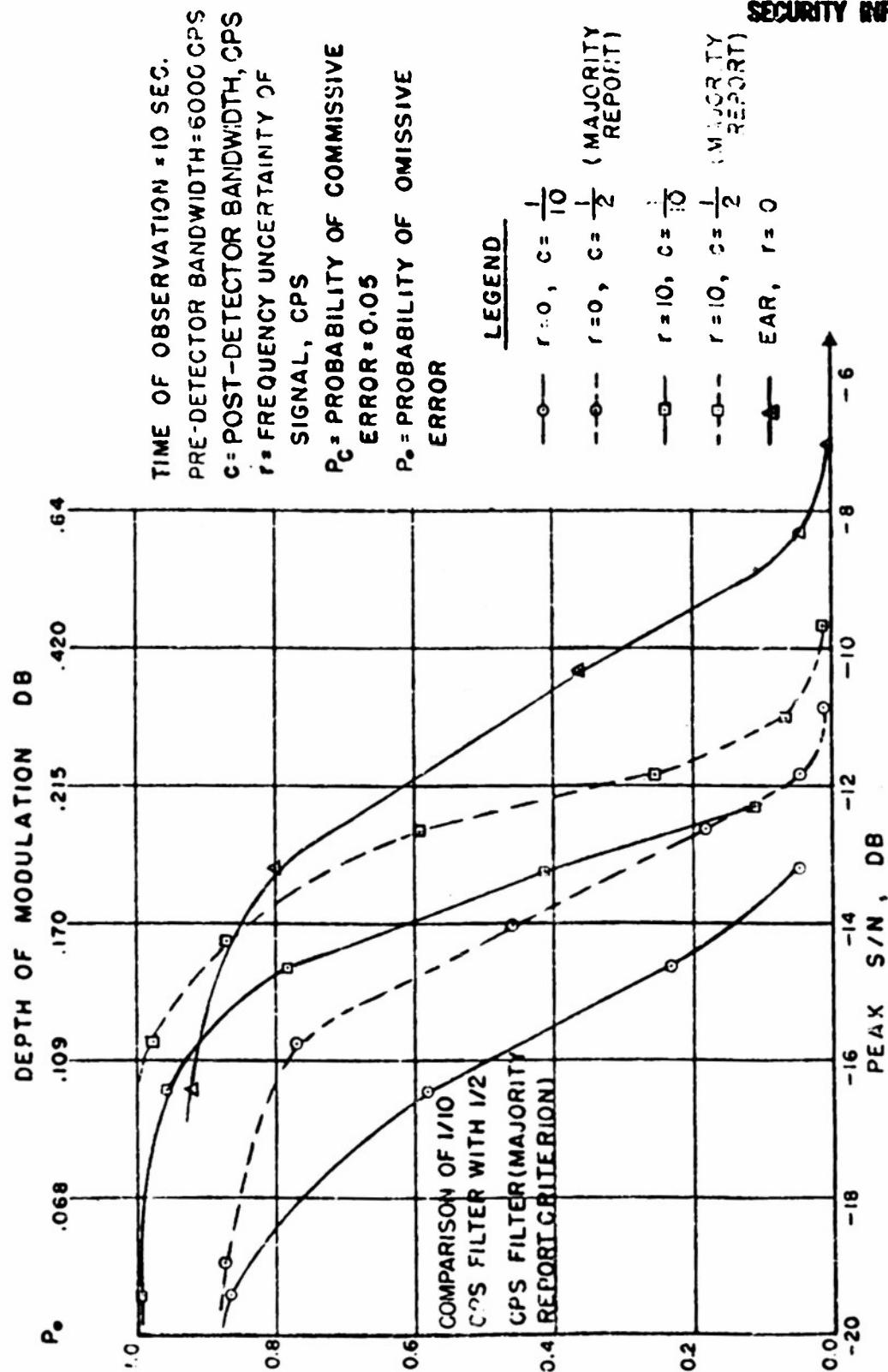


FIGURE 4| DETECTION OF AN AMPLITUDE MODULATED WHITE NOISE IN A WHITE NOISE BACKGROUND.

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